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Grading Of Metallic Starting Resistors For Electric Motors —With Examples.

By E. W. BRASS, Grad.I.E.E.

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GRADING OF METALLIC STARTING RESISTORS FOR ELECTRIC MOTORS — WITH EXAMPLES

By E. W. Brass

General Introduction.

The grading of starting resistors for electric motors is a subject which does not receive very extensive treatment in the normal text book. Furthermore, information is usually very scattered on the various types with the result that more than one source must be consulted before adequate information can be accumulated. It is hoped, therefore, that this article will provide an easy and concise means of reference.

Section 1 (*a*) will deal with some fundamental properties of D.C. motors. Starting resistors for three types of D.C. motors will be considered, viz., Shunt, Series and Compound ; these will be developed in sections 1 (*b*), 1 (*c*) respectively.

Section 2 will be devoted to the A.C. Slipring motor. This section will consist of :—

2 (*a*) Balanced Rotor Current Starting.
2 (*b*) Slip Regulators.
2 (*c*) Unbalanced Rotor Current Starting.

Section 3 deals with Primary resistor starters for the A.C. Squirrel Cage motor, although these are only infrequently used.

Necessity For a Starting Resistor.

The aim of a Starting Resistor is to apply the line voltage gradually to a motor. In its ideal state, the resistor would be cut out smoothly, a condition which can be assimilated by a liquid resistor, but is not practical with a solid one. Sufficient steps are therefore included in a solid resistor to keep the instantaneous peak currents and torques within reasonable limits.

SECTION 1—D.C. MOTORS.

1 (a) Fundamental Properties.

Resistor grading problems depend for their solution on a knowledge of the two following fundamental properties of D.C. motors.

(1) The back e.m.f. generated by a motor is proportional to the product of the flux and the speed.
(2) The torque exerted by the motor armature is proportional to the product of the flux and the armature current.

From (1) it follows that, for any given value of the speed, the back e.m.f. is proportional to the flux and vice versa. From (1) and (2) it follows that, for any given value of speed, the torque

is proportional to the product of the back e.m.f. and the armature current, since the back e.m.f. is proportional to the flux. In resistor design calculations we are not usually concerned with absolute values of the back e.m.f. flux, torque and speed, but rather with relative values expressed in terms of the values which obtain under normal full load conditions. The above two principles, together with the deductions there from, enable relative values of these quantities to be readily calculated. In some resistor grading problems a knowledge of the magnetization curve is necessary.

Starting Torque and Acceleration.

The value of the starting torque required from a motor will depend upon its duty. To start against a torque equal to that on full load, the motor must develop more than full load torque in order to accelerate the armature and the connected load. The starting torque may vary, according to duty, from about 50% to as much as 200% or more. From the fundamental properties of D.C. motors, it is evident that in the case of the constant flux machine (shunt motor) the armature current is a direct measure of the torque. In the particular case of a machine starting up against full load torque, the starting current will be about 125% to 150% of full load current. The initial resistance of the armature circuit is determined by the starting conditions. Thus, if

V = Voltage of the supply.
I = Starting current.
R_1 = Resistance of armature circuit.
Then $R_1 = V/I$.

It should be borne in mind, however, that this resistance includes the motor armature, brushes, interpoles, etc., and the series field in the case of the series or compound motor. As the motor speeds up, back e.m.f. is generated, the current falls and with it the developed torque. If the resistance of the armature circuit remained at R_1, stable conditions of speed would soon obtain when the current had fallen to a value such that the torque developed balanced that opposing. With the large external resistance necessary to limit the starting current, this steady speed would be low. It therefore becomes necessary to reduce the external resistance in the armature circuit, step by step, until the full supply voltage is applied to the armature. A tapped resistor fulfils this requirement, but it must be so graded that "Notching Up" can take place without abnormal current surges.

SECTION 1 (b)—D.C. SHUNT MOTOR.

When the armature of a D.C. motor is rotating, with the field excited, an e.m.f. is induced in it. The applied voltage is balanced by two components, namely the volts drop in the

armature circuit and the induced e.m.f. This latter is termed the "back e.m.f." This is shown diagrammatically in Fig. 1 (*a*). The back e.m.f. is proportional to the product of the flux and speed, and the volts drop in the armature circuit is proportional to the product of the current in and the resistance of the armature circuit.

Let V	= Line volts.
E_1, E_2, E_3, etc.	= Back e.m.f. when resistance R_1, R_2, R_3, etc., is in circuit.
I_s	= Lower value of armature current at which notching up is assumed to take place (usually the full load current of the motor).
I_{p2}, I_{p3}, I_{p4}, etc.	= Peak current when resistance is reduced to R_2, R_3, R_4, etc.
I	= Initial current on 1st notch.
R_1	= Total initial resistance of the armature circuit.
r_m	= Resistance of motor only (see appendix).

The current/time diagram relating to the currents in the above nomenclature is shown in Fig. 1 (*b*).

When the armature is rotating at a steady speed

$$V = E_1 + I_s R_1 \qquad (1 \cdot 1)$$

If the resistance of the armature circuit is now suddenly reduced to R_2, the motor speed remains unaltered for a brief instant; with constant field flux, the back e.m.f. will remain at its original value E_1. The armature current will however rise, momentarily, to I_{p2}, whence :—

$$V = E_1 + I_{p2} R_2 \qquad (1 \cdot 2)$$

From $(1 \cdot 1)$ $I_s = \dfrac{V - E_1}{R_1}$

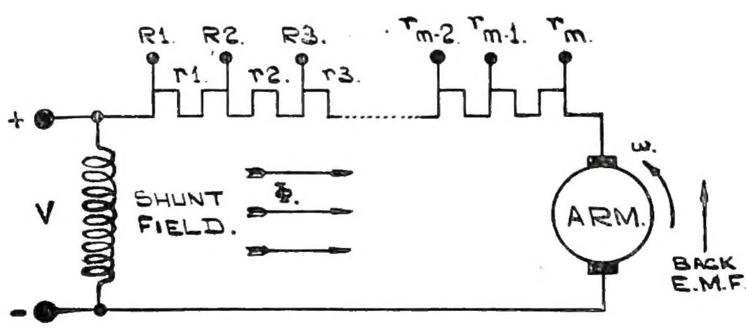


Fig. 1.—D.C. shunt motor with external armature resistance.

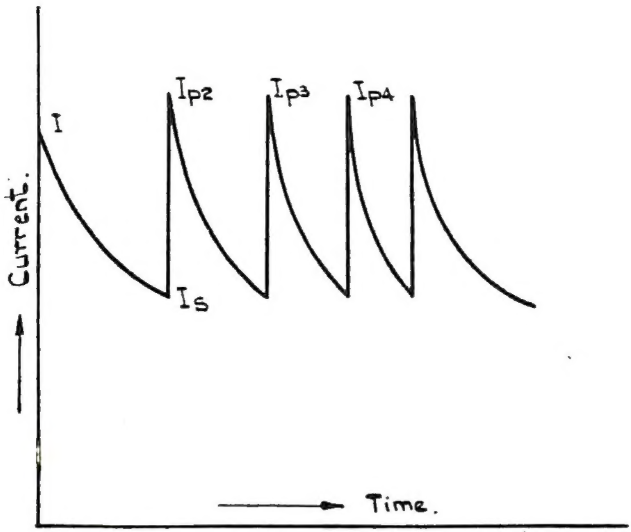


Fig. 1 (b).—Current/Time Diagram.

$$(1{\cdot}2)\quad I_{P2} = \frac{V-E_1}{R_2}$$

and the ratio $\frac{I_{P2}}{I_s} = \frac{V-E_1}{R_2} \times \frac{R_1}{V-E_1} = \frac{R_1}{R_2}$ (1·3)

This peak current occurs every time the external resistance in the armature circuit is reduced. It is usually of short duration and decreases to the value I_s when the motor has attained a steady speed. It is evident from (1·3) that the value of the peak current is influenced by the ratio $\frac{R_1}{R_2}$. In order to preserve the well-being of the control apparatus and motor, these peak currents must be restricted, and thus the ratio $\frac{R_1}{R_2}$ should not exceed a certain value. When the motor has reached a steady speed with the resistance of the armature circuit now R_2, the back e.m.f. will rise to E_2, so that

$$V = E_2 + R_2 I_s \tag{1·4}$$

If now the resistance of the armature circuit be further reduced to R_3 the back e.m.f. will remain momentarily at E_2, whence :—

$$V = E_2 + I_{p3}R_3 \qquad (1.5$$

From (1·4) $I_s = \dfrac{V-E_2}{R_2}$

(1·5) $I_{p3} = \dfrac{V-E_2}{R_3}$

and the ratio $\dfrac{I_{p3}}{I_s} = \dfrac{V-E_2}{R_3} \times \dfrac{R_2}{V-E_2} = \dfrac{R_2}{R_3}$ (1·6)

If $I_{p2} = I_{p3}$, i.e., equal peak currents, it follows that $\dfrac{R_1}{R_2} = \dfrac{R_2}{R_3}$

It is a usual criteria for starting resistors that all the peak currents should be the same, and successive application of the above will yield :

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} \; \ldots\ldots = \frac{r_{m-2}}{r_{m-1}} = \frac{r_{m-1}}{r_m} = K \qquad (1.7)$$

The series have a common ratio K and are thus in Geometric Progression (G.P.). If there are $n+1$ positions on the starting resistor, there will be n sections and since

$$\frac{\text{Peak currents}}{I_s} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} \; \ldots\ldots = \frac{r_{m-1}}{r_m}$$

$$\text{Then } K^n = \frac{R_1}{R_2} \times \frac{R_2}{R_3} \times \frac{R_3}{R_4} \; \ldots\ldots \times \frac{r_{m-2}}{r_{m-1}} \times \frac{r_{m-1}}{r_m}$$

$$\text{whence } K^n = \frac{R_1}{r_m}$$

i.e.,
$$K = \sqrt[n]{\frac{R_1}{r_m}} \qquad (1.8)$$

The value of R_1 is determined by the initial starting conditions. Thus, if the starting current, when the armature of the motor is stationary, is I

$$R_1 = \frac{V}{I} \qquad (1.9)$$

The use of the derived eq. (1.8) will be illustrated by some examples. The computed values are to slide-rule accuracy only.

Example 1.

A starting resistor having five sections is required for a 40 H.P. 440 volt shunt motor. The efficiency of the motor is 90% and full load current is to be passed on the 1st notch. Take $r_m = 0{\cdot}05 \times$ V/full load current. Assume I_s = full load current.

$$\text{Full load current} = \frac{40 \times 746}{440 \times 0{\cdot}9} = 75{\cdot}3 \text{ amps}$$

$$r_m = \frac{0{\cdot}05 \times V}{I_{FL}} = 0{\cdot}293 \text{ ohms}$$

$$R_1 = \frac{V}{I_{FL}} = \frac{440}{75{\cdot}3} = 5{\cdot}85 \text{ ohms}$$

$$\text{and } \frac{R_1}{r_m} = \frac{V}{I_{FL}} \times \frac{I_{FL}}{0{\cdot}05V} = 20$$

$$\text{From equation (1·8) } K = \sqrt[n]{\frac{R_1}{r_m}} = \sqrt[5]{20} = 1{\cdot}82$$

Whence

$$R_1 = \frac{V}{I} = \frac{440}{75{\cdot}3} = 5{\cdot}85 \text{ ohms}$$

$$R_2 = R_1 \div K = 5{\cdot}85 \div 1{\cdot}82 = 3{\cdot}22 \text{ ohms}$$

$$R_3 = R_2 \div K = 3{\cdot}2 \div 1{\cdot}82 = 1{\cdot}77 \text{ ohms}$$

$$R_4 = R_3 \div K = 1{\cdot}77 \div 1{\cdot}82 = 0{\cdot}972 \text{ ohms}$$

$$R_5 = R_4 \div K = 0{\cdot}972 \div 1{\cdot}82 = 0{\cdot}534 \text{ ohms}$$

$$r_m = R_5 \div K = 0{\cdot}534 \div 1{\cdot}82 = 0{\cdot}293 \text{ ohms}$$

Sectional Resistance

$r_1 = 2{\cdot}63$ ohms

$r_3 = 1{\cdot}45$ ohms

$r_3 = 0{\cdot}798$ ohms

$r_4 = 0{\cdot}438$ ohms

$r_5 = 0{\cdot}241$ ohms

Total 5·557 ohms

It should be noted that the sectional resistances are also in G.P. with the common ratio $K = 1{\cdot}82$. The total external resistance to be placed in series with the armature $= 5{\cdot}85 - 0{\cdot}293 = 5{\cdot}557$ ohms.

The current on the first notch is 75·3 amps and the peak currents on the remaining notches are $1{\cdot}82 \times 75{\cdot}3 = 137$ amps.

Example 2.

A 100 H.P. 440 volt shunt motor is to be started with peak currents not exceeding about twice full load current.

The starting current must be sufficient to accelerate against full load torque on the first notch of the resistor. Take $r_m = 0{\cdot}05 \times V$/full load current. Efficiency of motor 92%. Assume I_s = full load current.

$$\text{Full load current} = \frac{100 \times 746}{440 \times 0{\cdot}92} = 185 \text{ amps}$$

To accelerate against full load torque would require about 125 to 150% full load current. Allowing 150% full load current to pass on the 1st notch, armature stationary.

$$R_1 = \frac{440}{185 \times 1{\cdot}5} = 1{\cdot}6 \text{ ohms say}$$

$$r_m = \frac{0{\cdot}05 \times 440}{185} = 0{\cdot}119 \text{ ohms and } \frac{R_1}{r_m} = 13{\cdot}5$$

From equation (1·8)

$$K = \sqrt[n]{\frac{R_1}{r_m}}$$

and if K is not to exceed about 2, then

$$2^n = 13{\cdot}5 \text{ whence } n = 3{\cdot}76$$

The resistor must therefore have four sections

$$\therefore \quad \text{modified } K = \sqrt[4]{13{\cdot}5} = 1{\cdot}917$$

and *Sectional Resistance.*

$$R_1 = \frac{V}{I} = \frac{440}{185 \times 1{\cdot}5} = 1{\cdot}6 \text{ ohms}$$

	$r_1 = 0{\cdot}765$ ohms
$R_2 = R_1 \div K = 1{\cdot}6 \div 1{\cdot}917 = 0{\cdot}835$ ohms	$r_2 = 0{\cdot}400$ ohms
$R_3 = R_2 \div K = 0{\cdot}835 \div 1{\cdot}917 = 0{\cdot}435$ ohms	$r_3 = 0{\cdot}208$ ohms
$R_4 = R_3 \div K = 0{\cdot}435 \div 1{\cdot}917 = 0{\cdot}227$ ohms	$r_4 = 0{\cdot}108$ ohms
$r_m = R_4 \div K = 0{\cdot}227 \div 1{\cdot}917 = 0{\cdot}119$ ohms	
Total	1·481 ohms

The total external resistance to be placed in series with the armature is 1·6—0·119 = 1·481 ohms. Peak current on the first notch is 1·5 × 185 = 277·5 amps. and on the subsequent notches 1·917 × 185 = 354 amps. The sectional resistances are also in G.P. with a common ratio K = 1·917.

Since the resistance on the first notch is governed by the starting duty, it follows that the peak current on this notch is likewise fixed. In some cases it may be desirable to limit the initial current to less than full load. There are a number of reasons for this; for one thing, the first peak has to begin from zero, while the others nominally begin from full load, the operator being supposed to dwell long enough on each stud to permit the current to fall to this value. Further, static friction has to be overcome and backlash in gearing taken up at this point only. The effect upon the supply voltage may also limit the initial peak. However, it may be a specified condition that the peak currents on the first and subsequent notches must be the same. In order to meet this condition, equation (1.8) is modified thus.

Let K = Peak factor.

I_s = Steady or minimum value of accelerating current at which notching up is assumed to take place.

I = Initial current $= KI_s$.

Whence, from equation (1.8)

$$K = \sqrt[n]{\frac{R_1}{r_m}} = \sqrt[n]{\frac{V}{K\,I_s\,r_m}}$$

$$\therefore K = \sqrt[n+1]{\frac{V}{I_s\,r_m}} \qquad (1{\cdot}10)$$

Example 3.

A starting resistor having five sections and to give equal peak currents throughout is required for a 50 H.P. 220 volt shunt motor. The efficiency of the motor is 90%. Take $r_m = 0{\cdot}05 \times V$/Full load current. Assume I_s = full load current.

$$\text{Full load current} = \frac{50 \times 746}{220 \times 0{\cdot}9} = 189 \text{ amps.}$$

$$r_m = \frac{0{\cdot}05 \times 440}{189} = 0{\cdot}117 \text{ ohms}$$

$$\frac{V}{I_s\,r_m} = \frac{V}{189} \times \frac{189}{0{\cdot}05 \times V} = 20$$

From (1·10) $K^{5+1} = 20$. Whence $K = 1{\cdot}648$ and

	Sectional Resistance.
$R_1 = \dfrac{440}{189 \times 1{\cdot}648} = 1{\cdot}415$ ohms	
	$r_1 = 0{\cdot}555$ ohms
$R_2 = R_1 \div K = 1{\cdot}415 \div 1{\cdot}648 = 0{\cdot}86$ ohms	
	$r_2 = 0{\cdot}338$ ohms
$R_3 = R_2 \div K = 0{\cdot}86 \div 1{\cdot}648 = 0{\cdot}522$ ohms	
	$r_3 = 0{\cdot}205$ ohms
$R_4 = R_3 \div K = 0{\cdot}522 \div 1{\cdot}648 = 0{\cdot}317$ ohms	
	$r_4 = 0{\cdot}124$ ohms
$R_5 = R_4 \div K = 0{\cdot}317 \div 1{\cdot}648 = 0{\cdot}193$ ohms	
	$r_5 = 0{\cdot}076$ ohms
$r_m = R_5 \div K = 0{\cdot}193 \div 1{\cdot}648 = 0{\cdot}117$ ohms	
Total	1·298 ohms

The total external resistance to be placed in series with the armature $= 1{\cdot}415 - 0{\cdot}117 = 1{\cdot}298$ ohms. Peak current on first and subsequent notches $= 1{\cdot}648 \times 189 = 311$ amps.

Example 4.

A 75 H.P. 220 volt shunt motor is driving a load which can vary from 100% down to about 50%. A starting resistor is required to limit the peak currents to not greater than 150% full load current and sufficient notches are to be included to ensure that there is no appreciable "snatch" within the limits of the possible loading prescribed.

The efficiency of the motor is 92%. Take $r_m = 0{\cdot}05 \times V/$full load current.

Sufficient notches must be included on the controller to provide for any load between 50% and 100%. Since it would be impossible to cater for every possible load condition between 50% and 100%, it would probably be in order to provide for the 50%, 75% and 100% conditions, bearing in mind that the cost of the resistor and controller gear increase with the number of notches. The starting resistor specification is therefore :—

(*a*) To pass 50% full load current on first notch.
(*b*) To pass 75% full load current on second notch.
(*c*) To pass 100% full load current on third notch.

And thereafter peaks not to exceed 150% full load current.

$$\text{Full load current} = \frac{75 \times 746}{220 \times 0{\cdot}92} = 276 \text{ amps.}$$

$$\text{Total resistance on first notch} = R_1 = \frac{220}{276 \times 0{\cdot}5} = 1{\cdot}6 \text{ ohms say}$$

Total resistance on second notch $= R_2 = \dfrac{220}{276 \times 0{\cdot}75} = 1{\cdot}06$ ohms say

Total resistance on third notch $= R_3 = \dfrac{220}{276} = 0{\cdot}8$ ohms say

$$r_m = \frac{0{\cdot}05 \times 220}{276} = 0{\cdot}04 \text{ ohms}$$

and $\dfrac{R_3}{r_m} = \dfrac{0{\cdot}8}{0{\cdot}04} = 20$

If the peaks are not to exceed 150% then $1{\cdot}5^n = 20$
Whence $n = 7{\cdot}4$, say 8, giving a total of 10 sections.
Modified $K = \sqrt[8]{20} = 1{\cdot}455$

Whence :

		Sectional Resistance
R_1	$= 1{\cdot}6$ ohms	
		$r_1 = 0{\cdot}54$ ohms
R_2	$= 1{\cdot}06$ ohms	
		$r_2 = {\cdot}26$ ohms
$R_3 = \dfrac{220}{276}$	$= 0{\cdot}8$ ohms	
		$r_3 = 0{\cdot}25$ ohms
$R_4 = R_3 \div K$	$= 0{\cdot}55$ ohms	
		$r_4 = 0{\cdot}172$ ohms
$R_5 = R_4 \div K$	$= 0{\cdot}378$ ohms	
		$r_5 = 0{\cdot}118$ ohms
$R_6 = R_5 \div K$	$= 0{\cdot}26$ ohms	
		$r_6 = 0{\cdot}081$ ohms
$R_7 = R_6 \div K$	$= 0{\cdot}179$ ohms	
		$r_7 = 0{\cdot}056$ ohms
$R_8 = R_7 \div K$	$= 0{\cdot}123$ ohms	
		$r_8 = 0{\cdot}039$ ohms
$R_9 = R_8 \div K$	$= 0{\cdot}084$ ohms	
		$r_9 = 0{\cdot}026$ ohms
$R_{10} = R_9 \div K$	$= 0{\cdot}058$ ohms	
		$r_{10} = 0{\cdot}018$ ohms
$r_m = R_{10} \div K$	$= 0{\cdot}040$ ohms	
	Total	$1{\cdot}560$ ohms

Total external resistance to be placed in series with the armature $= 1{\cdot}6 - 0{\cdot}04 = 1{\cdot}56$ ohms.

The current on the first, second and third notches is 138, 207 and 276 amps respectively and the peak currents on the remaining notches (which are in G.P.) are $1{\cdot}455 \times 276 = 402$ amps. If the load torque on the first and second notch is greater than about 50% or 75% respectively, the motor would fail to start but would tend to do so on the third notch. The most arduous rating for sections one and two would be with the motor stalled and this condition would normally be allowed for in the actual resistor design.

From the preceding examples, it will be observed that when the total resistances R_1, R_2, R_3, etc., are in G.P., the sectional resistances are also in G.P. with the same constant ratio. This can easily be demonstrated as follows :—

$$K = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} \text{ etc.}$$

Therefore

$$K = \frac{R_1 - R_2}{R_2 - R_3} = \frac{R_2 - R_3}{R_3 - R_4}$$

Whence

$$r_1 = R_1 - R_2 = R_1\left(\frac{K-1}{K}\right) \qquad (1{\cdot}11)$$

$$\text{and } r_2 = \frac{r_1}{K} \text{ etc.}$$

SECTION 1 (c)—SERIES AND COMPOUND MOTORS.

In the series motor, the field is produced by the armature current traversing the field coils, which are in series with the armature (see Fig. (2)).

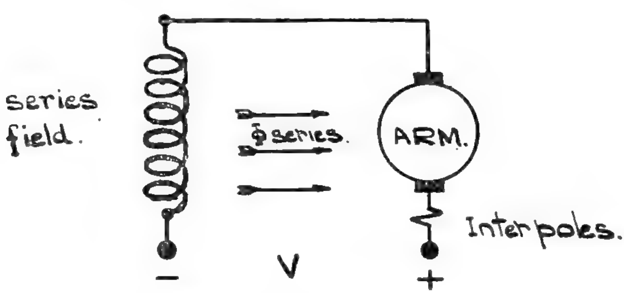


Fig. 2.—D.C. series motor.

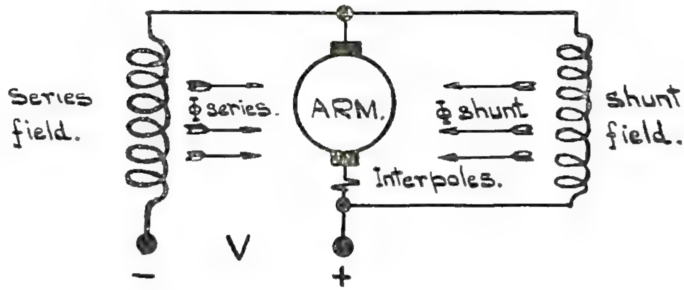


Fig. 3.—D.C. compound motor.

Neglecting the effect of armature reaction, the m.m.f. produced by the field therefore increases in direct proportion to the armature current. The value of the flux produced will, of course, vary according to the magnetization curve of the magnetic material constituting the flux path in the machine.

Fig. (3) shows a compound motor.

The field of this type of motor is excited by both shunt and series turns. The m.m.f. provided by the shunt turns is constant and the effect of the series turns has been described. Compound motors of ordinary design usually have about 80% shunt ampere turns and 20% series ampere turns. For other applications, where a series characteristic is mainly desired, a ratio of 80% series and 20% shunt ampere turns is used. This type is known as a series motor with a speed limiting winding.

Because of the dependency of the field flux upon the armature current, the grading of resistors for use with series or compound motors differ considerably from that of a shunt motor.

DERIVATION OF THE GRADING EQUATION.

Let V = Line volts.

E_1, E_2, E_3, etc. = Back e.m.f. when R_1, R_2, R_3, etc., is in circuit.

I_s = Lower value of armature current at which notching up is assumed to take place. (Usually the F.L. Current of the machine).

I = Initial current on first notch.

I_{p2}, I_{p3}, I_{p4}, etc. = Peak currents when resistance is reduced to R_2, R_3, R_4, etc.

r_m = Resistance of motor.

Φ_1 = Flux when I_s is flowing in the series coils.

Φ_2, Φ_3, Φ_4, etc. = Flux when I_{p2}, I_{p3}, I_{p4}, etc., is flowing in the series coils.

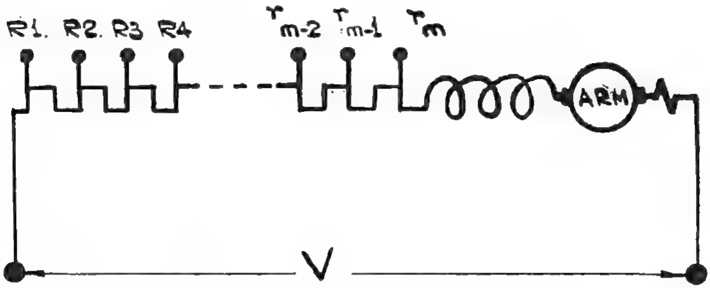


Fig. 4.—D.C. series motor with external resistance.

When the armature is rotating at a steady speed against a constant torque with I_s flowing in the series coils and R_1 in circuit.

$$V = E_1 + I_s\ R_1$$

$$\text{and } I_s = \frac{V - E_1}{R_1} \tag{1·12}$$

When the resistance is reduced to R_2, the current will rise to I_{p2} and the flux from Φ_1 to Φ_2, the motor speed remaining unaltered for a brief instant, whence

$$V = E_2 + I_{p2}\ .\ R_2 \text{ and}$$

since, at the same armature speed, $E_2 = \dfrac{\Phi_2}{\Phi_1}\ .\ E_1$

$$V = \frac{\Phi_2}{\Phi_1}\ .\ E_1 + I_{p2}\ .\ R_2$$

$$\text{and } I_{p2} = \frac{V - \dfrac{\Phi_2}{\Phi_1} E_1}{R_2}$$

$$\text{and the ratio of } \frac{I_{p2}}{I_s} = \frac{V - \dfrac{\Phi_2}{\Phi_1} E_1}{R_2} \times \frac{R_1}{V - E_1}$$

$$\text{Whence } R_2 = \frac{I_s}{I_{p2}} \times \left(V - \frac{\Phi_2}{\Phi_1} E_1 \right) \times \left(\frac{R_1}{V - E_1} \right)$$

Letting $\dfrac{I_s}{I_{p2}} = K$ and $\dfrac{\Phi_2}{\Phi_1} = K_1$, upon substituting in the above equation we obtain

$$R_2 = K\ (V-K_1E_1) \times \frac{R_1}{V-E_1}$$

From (1·12) $E_1 = V - I_sR_1$ whence

$$R_2 = K \left\{ \frac{V-K_1\ (V-I_sR_1)}{V-(V-I_sR_1)} \right\} \times R_1$$

$$= K \left\{ \frac{V-K_1\ (V-I_sR_1)}{I_sR_1} \right\} \times R_1$$

$$= \frac{KV}{I_s} - \frac{K.K_1\ V}{I_s} + \frac{K.K_1\ I_sR_1}{I_s}$$

$$= K\ K_1R_1 - K\ (K_1-1)\ \frac{V}{I_s} \qquad (1\cdot13)$$

If all the peak currents during starting are equal, then $I_{p2} = I_{p3} = I_{p4}$, etc., and successive application of the above procedure will yield.

$$R_1 = \frac{V}{I} \qquad (1\cdot13a)$$

$$R_2 = K\ K_1R_1 - K\ (K_1-1)\ \frac{V}{I_s} \qquad (1\cdot13b)$$

$$R_3 = K\ K_1R_2 - K\ (K_1-1)\ \frac{V}{I_s} \qquad (1\cdot13c)$$

$$R_4 = K\ K_1R_3 - K\ (K_1-1)\ \frac{V}{I_s} \qquad (1\cdot13d)$$

$$r_m = K\ K_1r_{m-1} - K\ (K_1-1)\ \frac{V}{I_s} \qquad (1\cdot13e)$$

It will be observed from the above series of equations that the ohmic resistance in circuit on each notch is a constant fraction (KK_1) of the ohmic resistance on the preceding notch minus a constant ohmic resistance $K\ (K_1-1)\ \frac{V}{I_s}$. The resistance R_1 is known from the specified starting conditions. Also known will be the resistance of the motor r_m. In the usual case the motor is assumed to start against a constant torque equal to full load from which it is evident that the minimum current on each notch is equal to the full-load current of the motor. Since both K and K_1

depend upon the ratio of the peak to the minimum current, the series of equations (1·13) cannot be solved directly. The usual procedure is to assign a reasonable value to K (i.e., to assume a likely peak current) from which, in consultation with the magnetization curves of the motor, K_1 can be fixed. From the specified starting duty R_1 can be obtained, and by substituting the values of K and K_1 in the series of equations (1·13), r_m must equal the ohmic resistance of the motor in the specified number of starting steps.

CHARACTERISTIC MAGNETIZATION CURVES.

Strictly speaking, starting resistors for series or compound motors should be designed from the characteristic magnetization curve of the motor in question. In practice, the resistor manufacturer rarely has access to such curves, consequently use must be made of a curve for the average motor. The only characteristic curves really necessary are those showing the relationship between the back e.m.f. generated at normal full load speed and the armature current. Table I gives the co-ordinates and Fig. (10) the plotted points for such a set of curves. With the almost universal use of commutating poles in modern D.C. machines, the effects of armature reaction can be ignored in resistor design calculations.

Example 5.

A starting resistor having 3 sections is required for a 40 H.P. 220 volt D.C. series motor. The efficiency of the motor is 90% and full load current is to be passed on the first notch.

Take $r_m = 0{\cdot}09 \times V/\text{full load current}$.

$$\text{Full load current} = \frac{40 \times 746}{220 \times 0{\cdot}9} = 151 \text{ amps.}$$

$$r_m = \frac{0{\cdot}09 \times 220}{151} = 0{\cdot}131 \text{ ohms}$$

Assuming the motor to be started against full load torque throughout, I_s will be 151 amps.

$$R_1 = \frac{220}{151} = 1{\cdot}46 \text{ ohms and } \frac{V}{I_s} = 1{\cdot}46$$

As a preliminary estimate let $\dfrac{I_p}{I_s} = 1{\cdot}7$

Whence $K = \frac{I_s}{I_p} = \frac{1}{1{\cdot}7}$ and from the characteristic curves

$$K_1 = \frac{103}{91} = 1{\cdot}13$$

$$K\,K_1 = \frac{1}{1{\cdot}7} \times 1{\cdot}13 = 0{\cdot}665 \text{ and } K\,(K_1 - 1)\frac{V}{I_s} = \frac{1}{1{\cdot}7}\ \ 1{\cdot}13 - 1)\ \ 1{\cdot}46$$
$$= 0{\cdot}112$$

Whence :

$$R_1 = \frac{V}{I_s} = \frac{220}{151} = 1{\cdot}46 \text{ ohms}$$

$$R_2 = 1{\cdot}46 \times 0{\cdot}665 - 0{\cdot}112 = 0{\cdot}858 \text{ ohms}$$
$$R_3 = 0{\cdot}858 \times 0{\cdot}665 - 0{\cdot}112 = 0{\cdot}458 \text{ ohms}$$
$$r_m = 0{\cdot}458 \times 0{\cdot}665 - 0{\cdot}112 = 0{\cdot}193 \text{ ohms}$$

Since the value of r_m resulting from the above is 0·193 ohms and the correct value should be 0·131 ohms, an adjustment of the assumed peak current is necessary. In order to reduce the value of 0·193 the assumed peak current will have to be bigger than that previously used.

Say $\frac{I_p}{I_s} = 1{\cdot}8$ whence $K = \frac{1}{1{\cdot}8}$

From the characteristic curve, $K_1 = \frac{104}{91} = 1{\cdot}144$

$$K\,K_1 = 0{\cdot}635 \text{ and } K\,(K_1 - 1)\ \frac{V}{I_s} = \frac{1}{1{\cdot}8}\ (1{\cdot}144 - 1)\ 1{\cdot}46 = 0{\cdot}117$$

Whence :

Sectional Resistance

$$R_1 = \frac{V}{I_s} = \frac{220}{151} = 1{\cdot}46 \text{ ohms}$$

$$R_2 = 1{\cdot}46 \times 0{\cdot}635 - 0{\cdot}117 = 0{\cdot}809 \text{ ohms} \qquad r_1 = 0{\cdot}651 \text{ ohms}$$

$$R_3 = 0{\cdot}809 \times 0{\cdot}635 - 0{\cdot}117 = 0{\cdot}397 \text{ ohms} \qquad r_2 = 0{\cdot}412 \text{ ohms}$$

$$r_m = 0{\cdot}397 \times 0{\cdot}635 - 0{\cdot}117 = 0{\cdot}135 \text{ ohms} \qquad r_3 = 0{\cdot}262 \text{ ohms}$$

Total 1·325 ohms

The value $r_m = 0{\cdot}135$ ohms accords sufficiently well with the correct value of 0·131 ohms and further adjustment of I_p is really unnecessary. The correct value of I_p is $1{\cdot}813 \times I_s$, but nothing will be gained by exploiting this example further. Assuming the value of $I_p = 1{\cdot}8$, the total external resistance to be placed in series with the armature $= 1{\cdot}46 - 0{\cdot}135 = 1{\cdot}325$ ohms. The current on the first notch is 151 amps and the peak currents on the subsequent notches are $1{\cdot}8 \times 151 = 272$ amps.

Example 6.

A starting resistor having 6 sections is required for a 100 H.P. 440 volt D.C. series motor. The efficiency of the motor is 92% and 75% full load current is to be passed on the first notch. Take $r_m = 0{\cdot}090$ V/Full load current.

$$\text{Full load current} = \frac{100 \times 746}{440 \times 0{\cdot}92} = 185 \text{ amps}$$

$$r_m = \frac{0{\cdot}09 \times 440}{185} = 0{\cdot}214 \text{ ohms}$$

Assuming the motor to be started against full load torque on the second and subsequent notches, I_s will be 185 amps.

$$R_1 = \frac{440}{185 \times 0{\cdot}75} = 3{\cdot}17 \text{ ohms and } \frac{V}{I_s} = \frac{440}{185} = 2{\cdot}38 \text{ ohms}$$

as a preliminary estimate let $\dfrac{I_p}{I_s} = 1{\cdot}4$

Whence $K = \dfrac{1}{1{\cdot}4}$ and from the characteristic curves $K_1 = \dfrac{100}{91}$

$$= 1{\cdot}1$$

$$K\,K_1 = \frac{1}{1{\cdot}4} \times 1{\cdot}1 = 0{\cdot}786 \text{ and } K\,(K_1 - 1)\frac{V}{I_s} = \frac{1}{1{\cdot}4}\;(1{\cdot}1 - 1)\;2{\cdot}38$$

$$= 0{\cdot}17$$

Whence :

$$R_1 = \frac{V}{I} = \frac{440}{185 \times 0{\cdot}75} = 3{\cdot}17 \text{ ohms}$$

$R_2 = 3{\cdot}17 \times 0{\cdot}786 - 0{\cdot}17 = 2{\cdot}32$ ohms

$R_3 = 2{\cdot}32 \times 0{\cdot}786 - 0{\cdot}17 = 1{\cdot}67$,,

$R_4 = 1{\cdot}67 \times 0{\cdot}786 - 0{\cdot}17 = 1{\cdot}14$,,

$R_5 = 1{\cdot}14 \times 0{\cdot}786 - 0{\cdot}17 = 0{\cdot}726$,,

$R_6 = 0{\cdot}726 \times 0{\cdot}786 - 0{\cdot}17 = 0{\cdot}4$,,

$r_m = 0{\cdot}4 \times 0{\cdot}786 - 0{\cdot}17 = 0{\cdot}144$,,

Since the value of $r_m = 0{\cdot}144$ calculated from the above is very different from the correct value of 0·214 ohms, a reassessment of I_p will be necessary. An observation will indicate that I_p has been estimated too high. Reducing I_p to, say, $1{\cdot}37 \times I_s$.

$$K = \frac{1}{1{\cdot}37} \text{ and } K_1 = \frac{99}{91} = 1{\cdot}09$$

$$K\,K_1 = \frac{1}{1{\cdot}37} \times 1{\cdot}09 = 0{\cdot}796 \text{ and } K\,(K_1 - 1)\,\frac{V}{I_s} = \frac{1}{1{\cdot}37}\;(1{\cdot}09 - 1)\;2{\cdot}38$$

$$= 0{\cdot}157$$

Whence :

Sectional Resistance

$$R_1 = \frac{V}{I} = \frac{440}{185 \times 0{\cdot}75} = 3{\cdot}17 \text{ ohms}$$

	$r_1 = 0{\cdot}807$ ohms
$R_2 = 3{\cdot}17 \times 0{\cdot}796 - 0{\cdot}157 = 2{\cdot}363$ ohms	
	$r_2 = 0{\cdot}640$,,
$R_3 = 2{\cdot}363 \times 0{\cdot}796 - 0{\cdot}157 = 1{\cdot}723$,,	
	$r_3 = 0{\cdot}510$,,
$R_4 = 1{\cdot}723 \times 0{\cdot}796 - 0{\cdot}157 = 1{\cdot}213$,,	
	$r_4 = 0{\cdot}405$,,
$R_5 = 1{\cdot}213 \times 0{\cdot}796 - 0{\cdot}157 = 0{\cdot}808$,,	
	$r_5 = 0{\cdot}323$,,
$R_6 = 0{\cdot}808 \times 0{\cdot}796 - 0{\cdot}157 = 0{\cdot}485$,,	
	$r_6 = 0{\cdot}256$,,
$r_m = 0{\cdot}485 \times 0{\cdot}796 - 0{\cdot}157 = 0{\cdot}229$,,	
Total	2·941 ohms

The value of $r_m = 0{\cdot}229$ accords reasonably well with the correct value of 0·214 and for practical purposes should be all right. The correct value of $I_p = 1{\cdot}376 \times I_s$. Assuming the value of $I_p = 1{\cdot}37 \times I_s$, the total external resistance to be placed in series with the armature is $3{\cdot}17 - 0{\cdot}229 = 2{\cdot}941$ ohms.

The current on the first notch is $0{\cdot}75 \times 185 = 139$ amps. The peak current on the second notch is $1{\cdot}37 \times 139 = 190$ amps and on subsequent notches, $1{\cdot}37 \times 185 = 253$ amps.

Whilst examples (5) and (6) are for the series motor, exactly the same procedure can be adapted for the compound motor, this time, of course, using the appropriate characteristic curve.

It will be noted that in the previous examples on the series motor, the sectional resistances are in G.P. having a common ratio $K.K_1$.

It is evident that with series motors, the number of steps necessary is less than for a shunt motor having the same value for the ratio I_p/I_s. Further more, it must be remembered that self induction will help considerably in preventing the current peaks from reaching the figures on which the previous calculations are made.

SECTION 2—A.C. SLIPRING MOTOR.

2 (a) Balanced Rotor Current Starting.

The polyphase induction motor has two basic parts, a stationary part called the PRIMARY or STATOR and a rotatable member called the SECONDARY or ROTOR. The primary, or stator, consists of a laminated core which is slotted and wound with a polyphase winding suitable for the supply system. The secondary, or rotor, consists of a cylindrical laminated core mounted on a shaft. The outer periphery of the core is slotted for the rotor winding, which, in the case of a slipring motor, is usually star connected and brought out to three sliprings mounted on the shaft. Under normal operating conditions these sliprings are short circuited. When the stator is energised, the polyphase stator winding produces a rotating magnetic field. This rotating field cuts both the stator and rotor windings. When the rotor is stationary, e.m.f.s of supply frequency are induced in it and the action is similar to that of a transformer, except that the field is rotating instead of alternating. When the rotor is rotating, the relative movement between the stator fields and rotor conductors is reduced, with a consequent reduction in both the rotor induced e.m.f. and frequency. The speed of the rotating field produced by the stator is constant and called SYNCHRONOUS SPEED. The relative cutting speed between the stator field and the rotor conductors can vary between SYNCHRONISM (when the rotor is stationary) and zero, when the rotor speed equals the speed of the rotating field. Actually, the rotor speed can never equal the speed of the rotating field, since there would be no change of flux linkages between the field

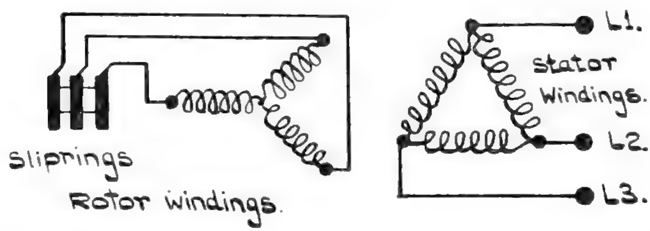


Fig. 5.—Schematic connections of A.C. slipring motor with delta connected stator and star connected rotor.

and the rotor conductors, resulting in no induced e.m.f. and current in the rotor. If the rotor shaft is not loaded, the machine has only to rotate itself against its mechanical losses ; the rotor speed will rise and approach very closely to the synchronous speed. The speed of rotation, n, of the rotor, is then less than the synchronous speed n_1 by the fraction s, known as the SLIP, whence,

$$s=\frac{n_1-n}{n_1}$$

On no load, the slip is generally less than about 1 per cent., rising to about 5 per cent. for the average motor on full load.

Torque Production.

With the rotor stationary and on open circuit, the stator winding excited from the supply system, e.m.f.s are induced in the rotor winding, which behaves in a similar manner as the secondary of a transformer. A voltmeter connected across any pair of sliprings would detect the secondary or rotor voltage, which, since the rotor is stationary, would alternate at the same frequency as that supplying the stator. If the sliprings are shorted, this induced e.m.f. will circulate a current in the rotor winding, the magnitude of this current being determined by the induced e.m.f. and the impedance of the rotor winding. The currents so produced interact with the rotating field of the stator to produce a torque. The magnitude of this torque can be shown to be proportional to rotor current × flux × cosine of the phase difference between the current and flux, i.e.

Torque $\propto \Phi \,.\, I_2 \,.\, \cos\phi_2$
where Φ = flux
I_2 = rotor current (Rms value)
ϕ_2 = the phase difference between the current in the rotor circuit and flux.

If the rotor is free to rotate and the torque produced exceeds the retarding torque, the rotor will accelerate in the same direction as the rotating field produced by the stator. As the rotor speed increases, the induced e.m.f. and frequency in the rotor circuit will decrease, owing to a decrease in the relative cutting speed of the rotating field and the rotor conductors. Stable speed conditions are obtained when the e.m.f. induced in the rotor winding is just sufficient to produce a current in the rotor circuit which creates a torque equal to the retarding torque.

Magnitude of Rotor Currents.

Let E_2 = Induced e.m.f. per phase at standstill.
r_m = Resistance of rotor winding per phase.

x_2 = Reactance of rotor winding per phase at standstill.
I_2 = Rotor current per phase.

Impedance of the rotor winding at standstill $= \sqrt{[r_m^2 + x_2^2]}$

Whence, rotor current per phase at standstill is

$$I_2 = \frac{E_2}{\sqrt{[r_m^2 + x_2^2]}}$$

When the rotor is running at slip s, the reactance becomes sx_2 and the impedance $\sqrt{[r_m^2 + s^2x_2^2]}$ whence

$$I_2 = \frac{s\,E_2}{\sqrt{[r_m^2 + s^2x_2^2]}} = \frac{E_2}{\sqrt{[(r_m/s)^2 + x_2^2]}} \qquad (2\cdot1)$$

The power factor of the rotor circuit at standstill is

$$\cos\phi_2 = \frac{r_m}{\sqrt{[r_m^2 + x_2^2]}} \qquad (2\cdot2)$$

and when running at slip s, $\cos\phi_2 = \dfrac{r_m}{\sqrt{[r_m^2 + s^2x_2^2]}}$ (2·3)

In the normal motor, the full load slip will be about 5% from which it is evident that the power factor of the rotor circuits is almost unity.

Magnitude of Generated Torque.

When the rotor is running with a slip s, corresponding to a rotor current I_2, the torque, assuming a constant flux Φ, will be

$$T = K\ I_2\Phi.\ \cos\phi_2$$

If R_R is the resistance of the rotor circuits, comprising that of the rotor winding and any added external resistance, then from (2·1)

$$I_2 = \frac{s\ E_2}{\sqrt{[R_R^2 + s^2\,x_2^2]}}$$

and from (2·3), $\cos\phi_2 = \dfrac{R_R}{\sqrt{[R_R^2 + s^2\,x_2^2]}}$

$$\text{Then, } T = K\,\Phi \left\{ \frac{s\,E_2}{\sqrt{[R_R^2 + s^2\,x_2^2]}} \times \frac{R_R}{\sqrt{[R_R^2 + s^2\,x_2^2]}} \right\}$$

$$= K\,\Phi\ .\ \frac{s\,E_2\,R_R}{R_R^2 + s^2\,x_2^2}$$

Hence, for constant supply frequency and p.d. in a given motor

$$T \propto \frac{s\ R_R}{R_R^2 + s^2\,x_2^2} \qquad (2\cdot4)$$

If R_R and s in the above expression were both increased, say M times, the value of the torque will remain unaltered. Thus if R_R is altered to MR_R, any value of torque originally obtained with a slip s is now obtained at a slip Ms. From this it is apparent that the speed of a slipring induction motor at a given torque can be controlled by inserting resistance in the rotor circuits. It should be noted, however, that only speed variation below synchronism can be effected in this manner.

Electrical Efficiency of the Rotor.

If E_2 is the voltage induced per phase in the rotor winding at standstill and I_2 the current at a power factor $\cos\phi_2$ then,

Power input to the rotor $P_2 = 3\ E_2 I_2 . \cos\phi_2$ (2·5)

When the rotor is running with a slip s, the current per phase of the rotor winding from (2·1) is

$$I_2 = \frac{s\,E_2}{\sqrt{[R_R^2 + s^2 x_2^2]}}$$

and the copper loss (assuming a 3 phase winding)

$$\text{is } P_R = 3\ R_R\ I_2^2$$
$$= 3\ R_R \cdot \frac{s^2 E_2^2}{R_R^2 + s^2 x_2^2}$$

now, from (2·3), $\cos\phi_2 = \dfrac{R_R}{\sqrt{[R_R^2 + s^2 x_2^2]}}$ whence, upon substituting in the above equation we obtain

$$P_R = 3 \cdot \frac{s^2\ E_2^2}{\sqrt{[R_R^2 + s^2 x_2^2]}} \cdot \cos\phi_2$$

but $I_2 = \dfrac{s\ E_2}{\sqrt{[R_R^2 + s^2 x_2^2]}}$ $\therefore$ $\underline{\underline{P_R = 3s\ E_2\ I_2 . \cos\phi_2}}$ (2·6)

Now, since the rotor input is P_2 and the rotor copper loss P_R the mechanical output $P_M = P_2 - P_R$

$$= 3E_2I_2 . \cos\phi_2 - 3s\ E_2\ I_2 . \cos\phi_2$$
$$= 3E_2I_2 . \cos\phi_2\ (1-s) \quad (2·7)$$

so that, the rotor efficiency is $\dfrac{P_M}{P_2} = \dfrac{3\ E_2I_2 . \cos\phi_2}{3\ E_2I_2 . \cos\phi_2}\ (1-s)$

$$\underline{\underline{= 1-s}} \quad (2·8)$$

Furthermore, $\dfrac{P_R}{P_M} = \dfrac{3s\ E_2I_2\cos\phi_2}{3\ E_2I_2\cos\phi_2\ (1-s)} = \dfrac{s}{1-s}$

and from (2·8) $\frac{P_M}{P_2} = 1 - s$ it follows that

$$\frac{P_R}{P_2} = \frac{s}{1-s} \cdot 1 - s = s$$

$$\text{i.e.,}\ P_R = sP_2 \tag{2·9}$$

It is evident that for a motor working against a load demanding a steady torque, the slip is proportional to the rotor copper loss, which, since the rotor current remains at a value consistent with the steady torque required, means that the slip is proportional to the resistance of the rotor circuits. This fact has been previously demonstrated.

Rotor Starting Resistors.

If full line voltage were applied to the stator of a slipring induction motor, with the rotor stationary and sliprings shorted, abnormally high currents would flow in both the stator and rotor circuits. Since the transference of energy from the stator to the rotor is essentially by transformer action, extra resistance inserted in the rotor circuits would have the effect of reducing both the rotor and stator currents. Furthermore, extra resistance inserted in the rotor circuits will increase the power factor of the rotor currents and thereby increase the starting torque. As the resistance of the rotor circuits is reduced, current peaks similar to those already described in the section on D.C. motors will occur. If these peaks are to be kept within certain limits (this is the essential feature of a starting resistor), the ratio of the successive rotor circuit resistances must likewise be fixed.

Grading of Rotor Starting Resistor.

Fig. (6) shows a typical arrangement.

The usual conditions are imposed in the following derivation of the grading equation. They are :

(*a*) The motor is assumed to start against a constant torque, usually the full load torque.

(*b*) The current during the starting period varies between the upper limit, I_2, and the lower limit (usually the full load current of the motor) I_1.

Let

E_2 = Rotor induced voltage per phase at standstill.

R_1, R_2, R_3, etc. = Total resistance of the rotor circuit per phase.

r_m = Resistance of the rotor windings per phase.

I_2 = Peak or upper limit of the current during starting.

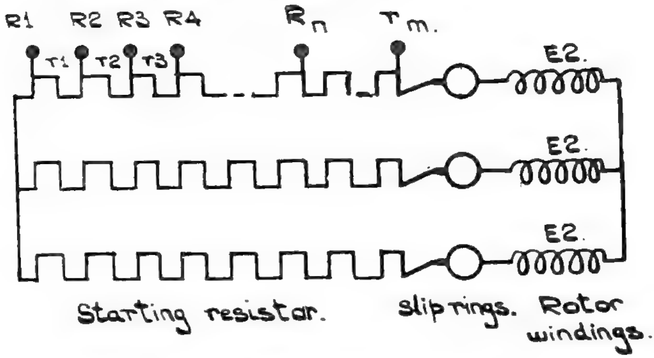


Fig. 6.—Rotor starting resistor.

I_1 = Lower limit of the rotor current at which notching up is assumed to take place.

s_2, s_3, s_4, etc. = Steady value of slips with R_1, R_2, R_3, etc., in circuit.

n = Number of resistance elements.

$n+1$ = Number of studs on the starter.

In the following derivations, the second form of equation (2·1) will be used. At the instant of making contact with the first stud, rotor stationary (i.e., $s_1=1$).

$$I_2 = \frac{E_2}{\sqrt{\left[\left(\frac{R_1}{s_1}\right)^2 + x_2^2\right]}}$$

As the rotor accelerates, the current will fall to I_1, and the slip to s_2, whence :

$$I_1 = \frac{E_2}{\sqrt{\left[\left(\frac{R_1}{s_2}\right)^2 + x_2^2\right]}}$$

At the instant of making contact with the second stud, the slip will remain momentarily at s_2, whence :

$$I_2 = \frac{E_2}{\sqrt{\left[\left(\frac{R_2}{s_2}\right)^2 + x_2^2\right]}}$$

As the rotor accelerates, the current will fall to I_1 and the slip to s_3, whence :

$$I_1 = \frac{E_2}{\sqrt{\left[\left(\frac{R_2}{s_3}\right)^2 + x_2^2\right]}}$$

From the above it is evident that on the nth stud

$$i_2 = \frac{E_2}{\sqrt{\left[\left(\frac{R_n}{s_n}\right)^2 + x_2^2\right]}}$$

$$I_1 = \frac{E_2}{\sqrt{\left[\left(\frac{R_n}{s_{n+1}}\right)^2 + x_2^2\right]}}$$

and on the last or $(n + 1)$th stud

$$I_2 = \frac{E_2}{\sqrt{\left[\left(\frac{r_m}{s_{n+1}}\right)^2 + x_2^2\right]}}$$

Thus $$I_2 = \frac{E_2}{\sqrt{\left[\left(\frac{R_1}{s_1}\right)^2 + x_2^2\right]}} = \frac{E_2}{\sqrt{\left[\left(\frac{R_2}{s_2}\right)^2 + x_2^2\right]}}$$

$$= - - - \frac{E_2}{\sqrt{\left[\left(\frac{R_n}{s_n}\right)^2 + x_2^2\right]}} = \frac{E_2}{\sqrt{\left[\left(\frac{r_m}{s_{n+1}}\right)^2 + x_2^2\right]}}$$

So that $$\frac{R_1}{s_1} = \frac{R_2}{s_2} = - \quad - \quad - \frac{R_n}{s_n} = \frac{r_m}{s_{n+1}}$$

$$\therefore \frac{s_1}{s_2} = \frac{R_1}{R_2}; \quad \frac{s_2}{s_3} = \frac{R_2}{R_3}; \quad - \quad - \quad \frac{s_n}{s_{n+1}} = \frac{R_n}{r_m} \qquad (2{\cdot}10)$$

also, from the equations for I_1, it follows that

$$\frac{R_1}{s_2} = \frac{R_2}{s_3} = - \quad - \quad - \quad - \quad - \quad = \frac{R_n}{s_{n+1}}$$

$$\therefore \frac{s_2}{s_3} = \frac{R_1}{R_2}; \quad \frac{s_3}{s_4} = \frac{R_2}{R_3}; \quad \frac{s_n}{s_{n+1}} = \frac{R_{n-1}}{R_n} \qquad (2{\cdot}11)$$

From (2·10) and (2·11) $\frac{s_1}{s_2}=\frac{s_2}{s_3}=\frac{R_1}{R_2}$; $\frac{s_2}{s_3}=\frac{s_3}{s_4}=\frac{R_2}{R_3}$; etc.

$$\text{Thus } \frac{s_1}{s_2}=\frac{s_2}{s_3}=\frac{s_3}{s_4}=\frac{s_n}{s_{n+1}}=k \qquad (2\cdot12)$$

$$\text{and } \frac{R_1}{R_2}=\frac{R_2}{R_3}=\frac{R_3}{R_4}=\frac{R_n}{r_m}=k \qquad (2\cdot13)$$

Thus, from (2.12) if there are n resistor elements

$$k^n=\frac{s_1}{s_{n-1}} \quad \text{i.e., } k=\sqrt[n]{\frac{1}{s_{n+1}}} \quad \text{Since } s_1=1 \qquad (2\cdot14)$$

Where s_{n+1} is the slip for the rotor running under normal conditions with the slip rings shorted and carrying I_2 amps. Under these conditions it is usual to assume that the slip is proportional to the rotor current. Thus, if the normal slip is, say, 3% and I_2 is twice the full load current of the rotor, then $s_{n+1}=6\%$.

From equation (2·13)

$$k^n=\frac{R_1}{r_m}$$

so that from (2·12) and (2·13), $\frac{R_1}{r_m}=\frac{s_1}{s_{n+1}}$

$$\text{Since } s_1=1 \text{ it follows that } R_1=\frac{r_m}{s_{n+1}} \qquad (2\cdot15)$$

When R_1 is calculated from equation (2·15)

$$k=\sqrt[n]{\frac{R_1}{r_m}} \qquad (2\cdot16)$$

The value of k, when calculated from equations (2·14) and (2·16) is the same, and the equations differ in form only. The total resistances have a common ratio k, and are thus in Geometric Progression (G.P.).

If the sectional resistances are r_1 r_2 r_3, etc., then

$$r_1=R_1-R_2=R_1-\frac{R_1}{k}=R_1\left(1-\frac{1}{k}\right)$$

$$\text{and } r_2=R_2-R_3=R_2-\frac{R_2}{k}=R_2\left(1-\frac{1}{k}\right)=\frac{1}{k}\cdot r_1$$

$$\text{and } r_3=\frac{1}{k}r_2,\ r_4=\frac{1}{k}r_3, \text{ etc.}$$

It is thus evident that the sectional resistances are also in G.P., with a common ratio k.

Current Peaks During Starting.

It is usual practice in resistance calculations to neglect the effects of x_2, the standstill reactance per phase of the rotor circuits. During starting, the initial resistance of the rotor circuits is generally much greater than x_2. As the external resistance is reduced the slip also decreases so that R/s always remains large compared with x_2.

$$\text{Hence } I_2 \simeq \frac{E_2}{R_1/s_1}$$

$$\text{and } I_1 \simeq \frac{E_2}{R_1/s_2}$$

$$\therefore \quad \frac{I_2}{I_1} = \frac{s_1}{R_1} \times \frac{R_1}{s_2} = \frac{s_1}{s_2} = k$$

This can be shown to apply throughout the various steps of the starting resistor.

It should be noted that the value of k calculated from equations (2·14) and (2·16), results in a rotor peak current on the first notch of magnitude $k \times$ full load rotor current. Specifications sometimes stipulate starting conditions whereby the initial current on the first notch differs from the peaks on the subsequent notches. In this case, R_1 is calculated from information given *re* the starting conditions and the value of k is thus computed from equation (2·16).

A slightly modified form of equation (2·14) is sometimes more convenient. It has been shown that the ratio of I_2/I_1 equals k when the rotor reactance is neglected. Since the slip is also approximately proportional to the rotor current it follows that $s_{n-1} = s_{FL} \times k$ where s_{FL} is the normal full load slip of the motor.

$$\text{Now from equation (2·14), } k = \sqrt[n]{\frac{1}{s_{n+1}}}$$

$$\therefore k = \sqrt[n]{\frac{1}{s_{FL}.k}}$$

$$\therefore k^n = \frac{1}{s_{FL}\,k} \quad \text{i.e.} \quad k^{n+1} = \frac{1}{s_{FL}} \quad \therefore k = \sqrt[n+1]{\frac{1}{s_{FL}}} \qquad (2·17)$$

The use of the derived equations (2·14) and (2·16) will be illustrated by some examples. The computed values are to slide-rule accuracy only.

Example 1.

A starting resistor having 5 sections and giving equal peaks on all notches is required for a 50 H.P. slip ring induction motor. The motor has a full load slip of 3% and the rotor resistance is 0·1 ohms per phase.

This is a direct application of equation (2·17).

$s_{FL} = 0{\cdot}03 \qquad n = 5 \qquad n+1 = 6$

$$k = \sqrt[n+1]{\frac{1}{0{\cdot}03}} = \sqrt[6]{\frac{1}{0{\cdot}03}} = \sqrt[6]{33{\cdot}4}$$

Using logs, $\log. k = \dfrac{\log 33{\cdot}4}{6} = \dfrac{1{\cdot}5237}{6} = 0{\cdot}25395$

$\therefore\ k = 1{\cdot}795$

From equation (2·15) $R_1 = \dfrac{r_m}{s_{n+1}} = \dfrac{0{\cdot}1}{1{\cdot}795 \times 0{\cdot}03} = 1{\cdot}86$ ohms say

whence

		Sectional Resistance
$R_1 =$	1·86 ohms	
		$r_1 = 0{\cdot}825$ ohms
$R_2 = 1{\cdot}86 \div 1{\cdot}795 = 1{\cdot}035$	ohms	
		$r_2 = 0{\cdot}457$ ohms
$R_3 = 1{\cdot}035 \div 1{\cdot}795 = 0{\cdot}578$	ohms	
		$r_3 = 0{\cdot}256$ ohms
$R_4 = 0{\cdot}578 \div 1{\cdot}795 = 0{\cdot}322$	ohms	
		$r_4 = 0{\cdot}1425$ ohms
$R_5 = 0{\cdot}322 \div 1{\cdot}795 = 0{\cdot}1795$	ohms	
		$r_5 = 0{\cdot}0795$ ohms
$r_m = 0{\cdot}1795 \div 1{\cdot}795 = 0{\cdot}10$	ohms	
		Total 1·7600 ohms

The total external resistance per phase to be inserted in the rotor circuits is 1·86—0·1 = 1·76 ohms. The peak currents on the first and subsequent notches is 1·795 times the full load current of the rotor.

Example 2.

What would be the number of resistance sections required to limit the current peaks to 1·5 times full load in the previous example?

From equation (2·17) $k = \sqrt[n+1]{\dfrac{1}{0{\cdot}03}}$

$$\therefore\ 1{\cdot}5 = \sqrt[n+1]{33{\cdot}4}$$

and $(n+1) \log. 1{\cdot}5 = \log. 33{\cdot}4$

$$\text{i.e. } (n+1) = 1{\cdot}5237 \div 0{\cdot}1761$$

whence $(n+1)=8{\cdot}67$, i.e., $n=8{\cdot}67-1=7{\cdot}67$

Since k must not exceed 1·5, n must not be less than 7·67, say, $n=8$ and modified value of $k={}^{9}\sqrt{33{\cdot}4}=1{\cdot}478$.

Example 3.

A starting resistor for a 100 H.P. slipring induction motor is required. The full load slip is 4% and the starting peak currents are not to exceed about twice full load current. Rotor volts between sliprings $=400$, rotor amps $=114$.

In this case it is first necessary to calculate the rotor resistance r_m.

From equation (2·6) the rotor I^2R loss is $P_R=3s\ E_2\ I_2 \cdot \cos\phi_2$ and, since $\cos\phi_2 \simeq 1$ then, $3\ I_2^2 r_m = 3s\ E_2\ I_2$

$$\text{i.e. } r_m = \frac{s\ E_2}{I_2}$$

substituting the appropriate values we obtain

$$r_m = \frac{0{\cdot}04 \times 400}{114 \times \sqrt{3}} = 0{\cdot}081 \text{ ohms}$$

From equation (2·17) $k = \sqrt[n+1]{\dfrac{1}{0{\cdot}04}}$

and since k is 2, $2={}^{n+1}\sqrt{25}$

from which $(n+1)=4{\cdot}65$, $\therefore\ n=3{\cdot}65$.

Taking the nearest integer we obtain $n=4$.

Modified value of $k={}^{5}\sqrt{25}=1{\cdot}903$

$$R_1 = \frac{r_m}{s_{n+1}} = \frac{0{\cdot}081}{1{\cdot}903 \times 0{\cdot}04} = 1{\cdot}06 \text{ ohms}$$

Whence

	Sectional Resistance
$R_1 = \qquad\qquad = 1{\cdot}06$ ohms	
	$r_1=0{\cdot}502$ ohms
$R_2=1{\cdot}06 \div 1{\cdot}903=0{\cdot}558$ ohms	
	$r_2=0{\cdot}265$ ohms
$R_3=0{\cdot}558 \div 1{\cdot}903=0{\cdot}293$ ohms	
	$r_3=0{\cdot}139$ ohms
$R_4=0{\cdot}293 \div 1{\cdot}903=0{\cdot}154$ ohms	
	$r_4=0{\cdot}073$ ohms
$r_m=0{\cdot}154 \div 1{\cdot}903=0{\cdot}081$ ohms	
Total	0·979 ohms

The total external resistance to be inserted in the rotor circuits is $1{\cdot}06-0{\cdot}081=0{\cdot}979$ ohms. The rotor peak currents on the first and subsequent notches are $1{\cdot}903\times114=217$ amps.

Neglecting rotor reactance, it is an easy task to compute the initial current on the first notch as follows.

$$I_2=\frac{400}{\sqrt{3}\times1{\cdot}06}=217 \text{ amps as above}$$

Where $\dfrac{400}{\sqrt{3}}$ is the rotor volts per phase and 1·06 ohms the rotor circuit resistance (strictly speaking impedance) per phase.

Example 4.

A starting resistor for a 75 H.P. slipring induction motor is required. The resistor design must be such that full load rotor current is passed on the first notch and the subsequent peak currents must not exceed 175% full load rotor current. Rotor volts between sliprings = 450. Full load slip = 4%.

Since the rotor current is not given, it must be estimated. For a slipring induction motor of normal design the overall rotor efficiency will be about 92%, so that

$$\text{Full load rotor amps}=\frac{\text{H.P.}\times746}{\sqrt{3}\times\text{RV}\times\eta\%}=\frac{75\times746}{\sqrt{3}\times450\times0{\cdot}92}$$
$$=78 \text{ amps.}$$

From the previous example, the rotor resistance per phase

$$r_m=\frac{\text{slip}\times\text{rotor volts}}{\text{Rotor amps}\times\sqrt{3}}=\frac{0{\cdot}04\times450}{78\times\sqrt{3}}=0{\cdot}133 \text{ ohms}$$

$$R_1=\frac{\text{rotor phase volts}}{\text{rotor current}}=\frac{450}{\sqrt{3}\times78}=3{\cdot}33 \text{ ohms}$$

$$\text{From equation (2.16) } k=\sqrt[n]{\frac{R_1}{r_m}}=\sqrt[n]{\frac{3{\cdot}33}{0{\cdot}133}}$$

Since k must not exceed 1·75,

$$\text{then } 1{\cdot}75=\sqrt[n]{\frac{3{\cdot}33}{0{\cdot}133}} \text{ from which } n=5{\cdot}76$$

Taking the nearest integer we obtain $n=6$,

$$\text{modified value of } k=\sqrt[6]{\frac{3{\cdot}33}{0{\cdot}133}}=1{\cdot}71$$

Whence

	Sectional Resistance
$R_1 = 3{\cdot}33$ ohms	$r_1 = 1{\cdot}38$ ohms
$R_2 = 3{\cdot}33 \div 1{\cdot}71 = 1{\cdot}95$ ohms	$r_2 = 0{\cdot}81$ ohms
$R_3 = 1{\cdot}95 \div 1{\cdot}71 = 1{\cdot}14$ ohms	$r_3 = 0{\cdot}472$ ohms
$R_4 = 1{\cdot}14 \div 1{\cdot}71 = 0{\cdot}668$ ohms	$r_4 = 0{\cdot}278$ ohms
$R_5 = 0{\cdot}668 \div 1{\cdot}71 = 0{\cdot}39$ ohms	$r_5 = 0{\cdot}161$ ohms
$R_6 = 0{\cdot}39 \div 1{\cdot}71 = 0{\cdot}229$ ohms	$r_6 = 0{\cdot}096$ ohms
$r_m = 0{\cdot}229 \div 1{\cdot}71 = 0{\cdot}133$ ohms	
	Total $3{\cdot}197$ ohms

The total external resistance to be inserted in the rotor circuits is $3{\cdot}33 - 0{\cdot}133 = 3{\cdot}197$ ohms. Rotor current on the first notch is 78 amps and the peak currents on the subsequent notches are $1{\cdot}71 \times 78 = 133{\cdot}5$ amps. The motor would only start on the first notch if the load torque was less than full load. Against full load torque, the motor would tend to start on the second notch. In some cases the load torque can vary considerably. In hoist motions of cranes, for example, depending upon the hook load, the torque can vary from say 50% to 100%. Special consideration must therefore be given to such cases, and due allowance made in the grading of the starting resistor. Again, some operations demand that the motor should run at a lower speed than normal against a fraction of the full load torque. This condition is termed "Intermittent Regulation with Creeping Speeds" and the usual adopted standard is one-fifth normal speed against a torque equal to one-third full load torque.

Example 5.

A starting resistor is required for a 50 H.P. slipring induction motor. The motor is driving a load which can vary from 50% to 100% and sufficient notches are to be provided on the controller so that there is no appreciable "snatch" within the prescribed possible loading limits. The maximum rotor peak currents must not exceed about 150% full load and the starting resistor must be designed to give a creep speed condition of 20% full speed against one-third full load torque. Rotor volts between sliprings = 350, rotor amps = 69. Full load slip = 5%.

The first notch on the resistor will provide for the creep speed condition. Since a possible load torque variation on subsequent

notches of between 50 to 100% must be catered for it would probably be in order to assume 50%, 75%, and 100% conditions. These would be obtained on the 2nd, 3rd and 4th notches respectively. Thereafter the peaks are restricted to 150% full load current, consequently the starting resistor specification is

(*a*) Creep speed conditions on the first notch.
(*b*) To pass 50% full load rotor current on the second notch.
(*c*) To pass 75% full load rotor current on the third notch.
(*d*) To pass 100% full load rotor current on the fourth notch.
(*e*) Therafter, rotor peak currents not to exceed about 150% full load current.

$$\text{Rotor resistance/phase } r_m = \frac{\text{slip} \times \text{rotor volts}}{\text{rotor current} \times \sqrt{3}} = \frac{0{\cdot}05 \times 350}{69 \times \sqrt{3}}$$
$$= 0{\cdot}146 \text{ ohms.}$$

Full load speed = 95% synchronous speed

$$\text{Slip at 20\% full load speed} = 1 - \frac{0{\cdot}95}{5} = 0{\cdot}81$$

Assuming $I_2 \propto$ torque, then current required at 1/3 full load torque = 69/3 = 23 amps.

Resistance per phase of rotor circuits for creep speed conditions

$$R_1 = \frac{0{\cdot}81 \times \text{rotor volts}}{\text{rotor current} \times \sqrt{3}} = \frac{0{\cdot}81 \times 350}{23 \times \sqrt{3}} = 7{\cdot}12 \text{ ohms}$$

Again, assuming rotor current $\propto$ torque, current required on the 2nd, 3rd and 4th notches would be $69 \times 0{\cdot}5$, $69 \times 0{\cdot}75$ and 69 amps respectively, so that,

$$R_2 = \frac{\text{rotor volts}}{\text{rotor amps} \times \sqrt{3}} \;\; (s = 1 \text{ since the motor is assumed not to start}) = \frac{350}{34{\cdot}5 \times \sqrt{3}}$$
$$= 5{\cdot}85 \text{ ohms}$$

$$R_3 = \frac{\text{rotor volts}}{\text{rotor amps} \times \sqrt{3}} \;\; (s = 1 \text{ since the motor is assumed not to start}) = \frac{350}{51{\cdot}75 \times \sqrt{3}}$$
$$= 3{\cdot}9 \text{ ohms}$$

$$R_4 = \frac{\text{rotor volts}}{\text{rotor amps} \times \sqrt{3}} \;\; (s = 1 \text{ since the motor is assumed not to start}) = \frac{350}{69 \times \sqrt{3}}$$
$$= 2{\cdot}93 \text{ ohms}$$

On the fourth notch of the controller normal starting is assumed with current peaks not exceeding approximately 150% full load current.

Applying equation (2.16) $k = \sqrt[n]{\frac{R_1}{r_m}}$ where R_1 in this case is on the 4th controller notch and is nominated R_4.

i.e., $1{\cdot}5 = \sqrt[n]{\frac{2{\cdot}93}{0{\cdot}146}} = \sqrt[n]{20}$ whence $n = 7{\cdot}4$

Since the peak currents are not rigidly fixed at 150%, it would probably be in order to assume $n = 7$, whence modified value of $k = \sqrt[7]{20} = 1{\cdot}535$.

So that resistor details :—

		Sectional Resistance	
R_1 =	7·12 ohms	r_1	= 1·27 ohms
R_2 =	5·85 ohms	r_2	= 1·95 ohms
R_3 =	3·9 ohms	r_3	= 0·97 ohms
R_4 =	2·93 ohms	r_4	= 1·02 ohms
R_5	= 2·93 ÷ 1·535 = 1·91 ohms	r_5	= 0·67 ohms
R_6	= 1·91 ÷ 1·535 = 1·24 ohms	r_6	= 0·43 ohms
R_7	= 1·24 ÷ 1·535 = 0·81 ohms	r_7	= 0·284 ohms
R_8	= 0·81 ÷ 1·535 = 0·526 ohms	r_8	= 0·184 ohms
R_9	= 0·526 ÷ 1·535 = 0·342 ohms	r_9	= 0·119 ohms
R_{10}	= 0·342 ÷ 1·535 = 0·223 ohms	r_{10}	= 0·077 ohms
r_m	= 0·223 ÷ 1·535 = 0·146 ohms		
		Total	6·974 ohms

The total resistance to be inserted in the rotor circuits is 7·12—0·146 = 6·974 ohms. The rotor currents on the 1st, 2nd, 3rd and 4th notches are 23, 34·5, 51·75 and 69 amps respectively, and the peak currents on the subsequent notches are 1·535 × 69 = 106 amps. If the load torque on the first and second notch is greater than about 50% or 75% respectively, the motor would fail to start. A torque a little greater than 50% could be handled on the third notch and likewise a torque a little greater than 75% on the fourth notch. Conditions requiring 100% torque would probably require the controller to move to the fifth notch.

SECTION 2b—SLIP REGULATORS.

When a resistor connected in the rotor circuit is intended not only for starting but also for regulating the speed of the motor, it becomes a SLIP REGULATOR or CONTROLLER. Whereas starting resistors are usually short time rated, i.e., they are designed to sustain the rotor currents for a period equal to the starting time, slip regulating resistors must be capable of carrying the load current of the rotor continuously without overheating. When large variations in regulation are required the regulating resistor can also be used as a starting resistor, in which case the resistor steps must conform with the requirements laid down for starting resistors.

The simplicity with which slip regulation can be accomplished in this manner is probably its only recommendation. Against this there are three major disadvantages, viz. :

(1) It is only suitable for drives requiring constant torques at all speeds.

(2) The efficiency is reduced in almost the same proportion as the speed.

(3) Speed regulation can only be effected from full load speed downwards.

In some cases only a small reduction in the full load speed is required ; in this case the resistor may be connected permanently in the rotor circuit via the sliprings and a simple shorting device used. Where large variations in regulation may be required, the regulating resistor can also be used for starting purposes and a controller is necessary.

Equation (2·9) shows that for constant torque the slip is proportional to the rotor loss watts. This is easily visualized when it is remembered that for constant torque the input to the rotor is constant and any extra loss in the rotor circuits (occurring in an external resistance) would reduce the H.P. at the motor shaft. Since, for similar rotor currents, the torque developed by the rotor would remain the same, it follows that the reduced H.P. is exhibited as a change of speed.

Slip Regulation—Constant Torque.

From equation (2·9) :

$P_R = sP_2$

where P_R = Rotor loss.

s = Slip.

P_2 = Rotor input.

now $P_R = 3.I_2^2 \times$ Resistance of the rotor circuits, where I_2 = full load rotor current per phase.

and $P_2 = 3.E_2.I_2.\cos\phi_2$, where E_2 = induced voltage per phase in the rotor winding at standstill, and $\cos\phi_2$ the power factor of the rotor circuit.

$\therefore\ 3\ I_2^2 \times$ Resistance of the rotor circuits $= s\ E_2 I_2 \cos\phi_2$.

$\therefore$ Resistance of the rotor circuits $= \dfrac{s\ E_2}{I_2} \times \cos\phi_2 = \dfrac{s\ E_2}{I_2}$ since

$$\cos\phi \simeq 1 \qquad (2\cdot18)$$

Let s_{FL} = Fractional slip at full load with the sliprings shorted.

s_x = Fractional slip at full load speed x.

r_m = Resistance of the rotor windings per phase.

R_e = External resistance per phase.

R_T = Total resistance of the rotor circuits per phase $= (R_e + r_m)$.

From (2·18)

$$r_m = \frac{s_{FL}.E_2}{I_2}$$

$$R_T = \frac{s_x.E_2}{I_2}$$

$$\therefore\ R_e = R_T - r_m = \frac{s_x E_2}{I_2} - \frac{s_{FL} E_2}{I_2} = (s_x - s_{FL}) \cdot \frac{E_2}{I_2}$$

It is normal practice to refer to the standstill rotor voltage between sliprings, whence the above equation becomes :

$$R_e = \frac{(s_x - s_{FL})}{I_2} \cdot \frac{V_R}{\sqrt{3}} \qquad (2\cdot19)$$

where V_R = Standstill rotor volts between sliprings.

Example 6.

A 50 H.P. 6 pole slipring induction motor has a full load speed of 960 r.p.m. What value of external resistances is required to reduce the speed to 800 r.p.m. when running against the same torque? Rotor volts between sliprings = 300, full load rotor current = 80 amps.

For a 6 pole motor the synchronous speed $= \dfrac{60 \times f}{p} = \dfrac{60 \times 50}{3}$

$$= 1000 \text{ r.p.m.}$$

$$s_{FL} = \frac{1000 - 960}{1000} = 0\cdot04$$

and $s_x = \dfrac{1000 - 800}{1000} = 0{\cdot}2$

Whence, from equation (2·19) :

$$R_e = \frac{(0{\cdot}2 - 0{\cdot}04)}{80} \cdot \frac{300}{\sqrt{3}} = 0{\cdot}347 \text{ ohms}$$

check :—

$$\text{H.P. lost in external resistance} = \frac{3 \times 80^2 \times 0{\cdot}347}{746} = 8{\cdot}92$$

Rotor output = Rotor input — Rotor losses.

If P_m = Rotor output at 960 r.p.m.

P_{m1} = Rotor output at 800 r.p.m.

Then decrease in rotor output $= P_m - P_{m1} =$

$(3\,E_2I_2.\cos\phi_2 - 3s_{FL}\,E_2I_2.\cos\phi_2) - (3\,E_2I_2.\cos\phi_2 - 3s_x\,E_2I_2.\cos\phi_2)$

$= 3\,E_2I_2\cos\phi_2\,(1 - s_{FL} - 1 + s_x)$

Since $\cos\phi_2 \simeq 1$, then

$$\text{H.P. decrease} = 3 \times \frac{300}{\sqrt{3}} \times \frac{80\,({\cdot}20 - 0{\cdot}04)}{746} = 8{\cdot}92$$

Which checks with the loss in the external resistance.

Example 7.

What value of external resistance would be required for a similar regulation against 50% full load torque in the previous example ? Assume the rotor current to be proportional to torque.

Rotor current $= 0{\cdot}5 \times 80 = 40$ amps.

$$\text{Whence } R_e = \frac{(0{\cdot}2 - 0{\cdot}04)}{\sqrt{3}} \cdot \frac{300}{40} = 0{\cdot}694 \text{ ohms}$$

Example 8.

It is desired to regulate the speed of a 100 H.P., 4 pole, slipring induction motor, from half to full speed in 10 equal steps when running against constant full load torque. The full load speed of the motor is 1440 r.p.m. Standstill rotor volts between sliprings = 500, full load rotor current = 96 amps.

For a 4 pole motor the synchronous speed $= \dfrac{60 \times f}{p} = \dfrac{60 \times 50}{2}$

$= 1500$ r.p.m.

$$s_{FL} = \frac{1500 - 1440}{1500} = 0{\cdot}04$$

$$\text{Slip at half speed} = \frac{1500-720}{1500} = 0{\cdot}52$$

Change in slip required in 10 equal steps $= 0{\cdot}52 - 0{\cdot}04 = 0{\cdot}48$.

Change in slip per step $= 0{\cdot}48 \div 10 = 0{\cdot}048$.

From equation (2·19) :

$$R_e = \frac{(s_X - s_{FL})}{I_2} \quad \frac{V_R}{\sqrt{3}}$$

$$\text{Whence } R_e = \frac{(s_X - s_{FL})}{96} \cdot \frac{500}{\sqrt{3}} = (s_X - s_{FL}) \cdot 3$$

Which upon working out for the various speed steps becomes

Speed r.p.m.	$s_X - s_{FL}$	*External Rotor Resistance* R_e
720	0·48	1·44 ohms
792	0·432	1·296 ohms
864	0·384	1·152 ohms
936	0·336	1·008 ohms
1008	0·288	0·864 ohms
1080	0·240	0·720 ohms
1152	0·192	0·576 ohms
1224	0·144	0·432 ohms
1296	0·096	0·288 ohms
1368	0·048	0·144 ohms
1440	0	0

It would be interesting to check the suitability of the regulating resistor for starting purposes.

$$\text{Starting current on the lowest speed notch} = \frac{500}{\sqrt{3} \times (1{\cdot}44 + r_m)}$$

$$\text{Where } r_m = s_{FL} \quad \frac{E_2}{I_2} = \frac{0{\cdot}04 \times 500}{96 \times \sqrt{3}} = 0{\cdot}12 \text{ ohms}$$

$$\text{Whence, starting current} = \frac{500}{\sqrt{3} \times (1{\cdot}44 + 0{\cdot}12)} = 185 \text{ amps.}$$

This represents a rotor starting current of 192% full load and, unless some special conditions prevail, should be quite in order for the average duty. Of course, the peak currents throughout the starting cycle will not be equal, the minimum rotor peak current obtaining when moving from the lowest speed step and the highest when the last section of the regulator is short circuited. These having values of :—

$$\frac{(0{\cdot}48+0{\cdot}04)}{(0{\cdot}432+0{\cdot}04)}=110\% \text{ rotor full load current, minimum value}$$

and $$\frac{(0{\cdot}048+0{\cdot}04)}{0{\cdot}04}=220\% \text{ rotor full load current, maximum value.}$$

Slip Regulation With Variable Load Torque.

Some motors drive loads the torque of which varies with speed, e.g., fans and centrifugal pumps. In the case of fans, the accepted laws are :—

Horse Power $\propto$ (Speed)^3

Since, Horse Power $\propto$ Speed $\times$ Torque

Then, Torque $\propto$ (Speed)^2

Making the normal assumption that the rotor current is proportional to torque, the rotor current

$$=\left\{\frac{\text{Actual Speed}}{\text{Full Speed}}\right\}^2 \times \text{normal full load rotor current.}$$

Using the same nomenclature as the constant torque case, the full load speed $=(1-s_{FL})\times$ Synchronous speed, and the speed at step $x=(1-s_x)\times$ Synchronous speed.

Then, the rotor current I_2 is related by the expression

$$I_2=\left\{\frac{1-s_x}{1-s_{FL}}\right\}^2\times \text{full load rotor current}$$

Whence, equation (2·19) becomes modified for fan drives thus

$$R_e=\frac{(s_x-s_{FL})}{\left\{\dfrac{1-s_x}{1-s_{FL}}\right\}^2 .I_2}\times\frac{V_R}{\sqrt{3}} \qquad (2{\cdot}20)$$

Example 9.

A ventilating unit is driven by a 4 pole, 40 H.P. slipring motor, having a full load speed of 1440 r.p.m. What would be the value of external resistance, placed in the rotor circuits, to reduce the speed to half its maximum value against full load torque? Standstill rotor volts between sliprings $=350$, full load rotor current $=55$ amps.

For a 4 pole motor the synchronous speed $=1500$ r.p.m.

$$s_{FL}=\frac{1500-1440}{1500}=0{\cdot}04$$

$$\text{and } s_x = \frac{1500-720}{1500} = 0{\cdot}52$$

Whence, from (2·20) $R_e = \dfrac{(0{\cdot}52-0{\cdot}04)}{\left\{\dfrac{1-0{\cdot}52}{1-0{\cdot}04}\right\}^2 \times 55} \times \dfrac{350}{\sqrt{3}} = 7{\cdot}04$ ohms

Example 10.

What would be the values of the external resistances to be placed in the rotor circuits to regulate the speed of the motor in the previous example from 1440 r.p.m. down to 440 r.p.m. in increments of 200 r.p.m.?

The various details will be tabulated and equation (2·20) applied thus :—

Speed r.p.m.	$s_x - s_{FL}$	$1 - s_x$	$1 - s_{FL}$	*External Rotor Resistance* R_e
440	0·667	0·293	0·96	26·3 ohms
640	0·534	0·426	0·96	10 ohms
840	0·401	0·559	0·96	4·35 ohms
1040	0·267	0·693	0·96	1·89 ohms
1240	0·133	0·826	0·96	0·66 ohms
1440	0	0·96	0·96	0

In computing the resistance values for slip regulators, it should be remembered that one usually works from the full load rotor current. In general, a mechanism or drive requiring a certain H.P. will be driven by a motor which will possibly have a reserve capacity above that required. Consequently, the motor rotor current will be somewhat less than its full load value. Bearing this in mind, it is good practice to increase the maximum ohmic value of the resistor by say 15%, or, provide tappings at ±15%, ±10% and ±5% where critical speed control is required. Furthermore, if a constant speed at any particular controller setting is required, the resistor material should have a low or negligible temperature coefficient. Of course, this latter point does not apply to resistors used for starting purposes only, since the actual resistance values on any particular notch are not very critical.

SECTION 2(c)—UNBALANCED ROTOR CURRENT STARTING.

When the sections of an external resistor, connected in the rotor circuits, are short circuited in successive phases, unbalanced rotor currents will result. Such a medium of control used for starting purposes is termed unbalanced rotor current starting.

In general, motor starting in this manner results in a reduction in the number of resistor sections and control gear contacts for a similar performance as the balanced starting previously considered. There is no standard method of grading and cutting out resistance in this manner. Some schemes seem to be devised with the express purpose of reducing the number of control gear contacts for economic reasons. However, indiscriminate short circuiting of resistor sections without due recourse to design is liable to subject the motor to severe treatment. It can be stated (although some justification is required), that if the total resistances of each of the three rotor legs, bear to one another the relation of the geometric progression, the torque produced by the three unbalanced currents is the same as that produced under balanced conditions in which the three legs have the same resistance as the geometric mean ; thus, if R_1, R_2 and R_3 are the resistance of the three unbalanced legs of the rotor circuit, then the torque produced will be the same as three balanced legs each of resistance $\sqrt[3]{R_1 \times R_2 \times R_3}$

Further, if these three unbalanced legs are in G.P. with a constant ratio K, then

$$R_2 = \frac{R_1}{K} \text{ and } R_3 = \frac{R_2}{K} = \frac{R_1}{K^2}$$

whence, the geometric mean $= \sqrt[3]{R_1 \times \frac{R_1}{K} \times \frac{R_1}{K^2}} = \frac{R_1}{K} = R_2$

where R_2 is the leg which has the intermediate ohmic value, i.e., if three phases have ohmic values of 8, 4 and 2 ohms, then the 4 ohm leg has the intermediate ohmic value and is usually termed the "middle" leg. It will be appreciated, of course, that the "middle" leg alters on every notch ; this should become more evident as progress is made.

Control Schemes.

Schemes of control for four to eight steps are shown in Fig. (7). The figures opposite the contactors for shorting out the resistor sections indicate the order of closing and also the step on which they are closed. The first step or notch is represented by the switching on of the stator, the final step cutting out simultaneously the resistance remaining in two of the legs. Although unbalanced rotor currents of magnitude greater than the normal full-load current will occur during starting, there is little likelihood of damage due to overheating as they are changed round at every step. Again, whereas the starting time of the motor may be a minute or so (even seconds in some cases), the temperature-time constant of the machine will probably be of the order of several minutes, say at least 30 for even the smallest motor requiring rotor starting.

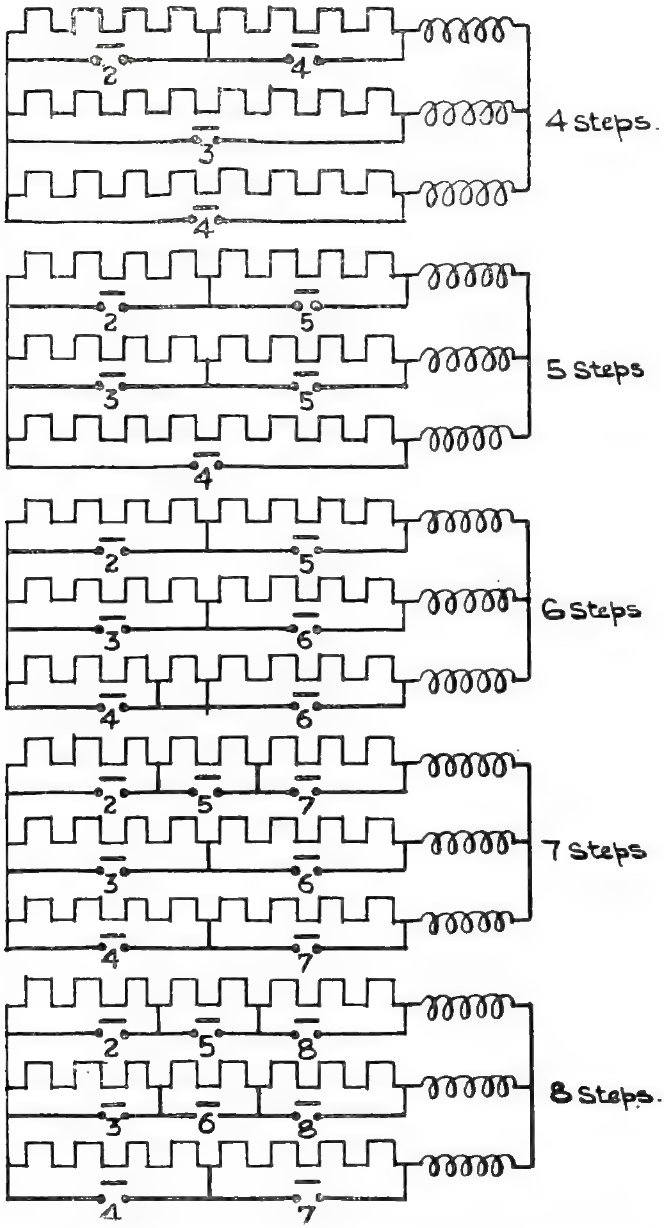


Fig. 7.—Control schemes for unbalanced rotor current starting.

Derivation of the General Equations—Neglecting Rotor Reactance.

It is necessary to derive expressions so that the currents in each of the unbalanced rotor legs can be computed. For this purpose, it will be assumed that the rotor windings and external resistor are star connected as indicated in Fig. (8).

Let V = Standstill rotor volts between sliprings.

E = Standstill rotor volts per phase $= V/\sqrt{3}$

s = Slip

E_1 = Phase volts of phase 1 $= sE\ (1+j0)$ (reference).

E_2 = Phase volts of phase 2 $= sE\ (-0{\cdot}5-j\ \sqrt{3}/2)$.

E_3 = Phase volts of phase 3 $= sE\ (-0{\cdot}5+j\ \sqrt{3}/2)$

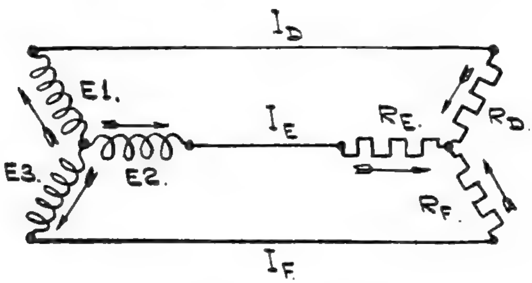


Fig. 8.—Pertaining to unbalanced rotor current starting.

Where E_1, E_2 and E_3 are vector quantities assuming counter clockwise phase rotation.

Let R_D = Total circuit resistance of phase 1.

R_E = Total circuit resistance of phase 2.

R_F = Total circuit resistance of phase 3.

I_D, I_E, I_F = Respective values of instantaneous currents.

Adopting the convention that e.m.f.s and currents directed away from the neutral point are positive we have,

$$E_1 - E_2 = I_D\ R_D - I_E\ R_E \qquad (a)$$

$$E_2 - E_3 = I_E\ R_E - I_F\ R_F \qquad (b)$$

$$E_1 - E_3 = I_D\ R_D - I_F\ R_F \qquad (c)$$

Applying Kirchoff's first law to the neutral point of the rotor winding, whence,

$$I_D + I_E + I_F = 0 \qquad (d)$$

Eliminating I_E and I_F from the above expressions to find I_D

$$\frac{E_1 - E_2}{R_E} = I_D\ \frac{R_D}{R_E} - I_E \qquad \text{from } (a)$$

$$\frac{E_1 - E_3}{R_F} = I_D \quad \frac{R_D}{R_F} - I_F \qquad \text{from } (c)$$

$$0 = I_D \qquad + I_E + I_F \qquad \text{from } (d)$$

adding,

$$\frac{E_1 - E_2}{R_E} + \frac{E_1 - E_3}{R_F} = I_D \left(1 + \frac{R_D}{R_E} + \frac{R_D}{R_F} \right)$$

Whence,

$$I_D = \frac{E_1 - E_2}{R_E \left(1 + \frac{R_D}{R_E} + \frac{R_D}{R_F} \right)} + \frac{E1 - E3}{R_F \left(1 + \frac{R_D}{R_E} + \frac{R_D}{R_F} \right)}$$

$$= \frac{E_1 - E_2}{R_E + R_D + \frac{R_D R_E}{R_F}} + \frac{E_1 - E_3}{R_F + \frac{R_D R_F}{R_E} + R_D}$$

$$= \frac{R_F (E_1 - E_2)}{R_E R_F + R_D R_F + R_D R_E} + \frac{R_E (E_1 - E_3)}{R_E R_F + R_D R_F + R_D R_E}$$

and

$$I_D (R_E R_F + R_D R_F + R_D R_E) = (R_E + R_F) E_1 - R_F E_2 - R_E E_3 \qquad (2{\cdot}21)$$

Likewise,

$$I_E (R_E R_F + R_D R_F + R_D R_E) = (R_D + R_F) E_2 - R_F E_1 - R_D E_3 \qquad (2{\cdot}22)$$

$$I_F (R_E R_F + R_D R_F + R_D R_E) = (R_D + R_E) E_3 - R_E E_1 - R_D E_2 \qquad (2{\cdot}23)$$

Letting

$k_R = (R_E R_F + R_D R_F + R_D R_E)$ and substituting for E_1, E_2 and E_3 in (2·21) we get,

$$I_D k_R = (R_E + R_F).sE (1 + j0) - R_F.sE (-0{\cdot}5 - j\sqrt{3}/2) - R_E.sE (-0{\cdot}5 + j\sqrt{3}/2)$$

dividing by sE and multiplying out

$$\frac{I_D k_R}{sE} = (R_E + R_F) + 0{\cdot}5\, R_F + j\sqrt{3}/2\, R_F + 0{\cdot}5\, R_E - j\sqrt{3}/2\, R_E$$

$$= \frac{3}{2} (R_E + R_F) + j \frac{\sqrt{3}}{2} (R_F - R_E)$$

$$\text{and } I_D = \frac{sV}{\sqrt{3}} \cdot \frac{1}{k_R} \left\{ \frac{3}{2} (R_E + R_F) + j \frac{\sqrt{3}}{2} (R_F - R_E) \right\} \qquad (2{\cdot}24)$$

Likewise,

$$I_E = \frac{sV}{\sqrt{3}} \cdot \frac{1}{k_R} \left\{ \frac{3}{2} (R_D + R_F) + j \frac{\sqrt{3}}{2} (R_D - R_F) \right\} \qquad (2{\cdot}25)$$

and $$I_F = \frac{sV}{\sqrt{3}} \cdot \frac{1}{k_R} \left\{ \frac{3}{2} (R_D + R_E) + j \frac{\sqrt{3}}{2} (R_E - R_D) \right\} \tag{2·26}$$

Now equations (2·24), (2·25) and (2·26) are the vector forms of the currents I_D, I_E and I_F respectively, and the absolute values are :—

$$I_D = \frac{sV}{\sqrt{3}} \cdot \frac{1}{k_R} \cdot \sqrt{\left[\left\{ \frac{3}{2} (R_E + R_F) \right\}^2 + \left\{ \frac{\sqrt{3}}{2} (R_F - R_E) \right\}^2 \right]}$$

$$= \frac{sV}{\sqrt{3}} \cdot \frac{1}{k_R} \cdot \sqrt{\left[\frac{9}{4} (R_E^2 + 2R_E\ R_F + R_F^2) + \frac{3}{4} (R_F^2 - 2R_F\ R_E + R_E^2) \right]}$$

$$= \frac{sV}{\sqrt{3}} \cdot \frac{1}{k_R} \cdot \sqrt{\left[\frac{12}{4} R_E^2 + \frac{12}{4} R_E\ R_F + \frac{12}{4} R_E^2 \right]}$$

$$= \frac{sV}{k_R} \cdot \sqrt{[R_E^2 + R_E\ R_F + R_F^2]} \tag{2·27}$$

Likewise

$$I_E = \frac{sV}{k_R} \cdot \sqrt{[R_D^2 + R_D\ R_F + R_F^2]} \tag{2·28}$$

and

$$I_F = \frac{sV}{k_R} \cdot \sqrt{[R_D^2 + R_D\ R_E + R_E^2]} \tag{2·29}$$

An examination of equations (2·24), (2·25) and (2·26) will reveal that the currents are composed of in phase and quadrature components. This is due to the shifting of the rotor electrical load neutral point, caused by the unbalanced rotor currents. Effectively, the rotor currents have been displaced from the rotating gap flux set up by the stator. The rotor currents which are effective in creating torque are those in phase with the flux. These currents are represented by the "real" parts of equations (2·24), (2·25) and (2·26) respectively. In the case of I_D this can be demonstrated as follows :—

Torque producing component of

$$I_D = \frac{sV}{k_R} \cdot \sqrt{[R_E^2 + R_E\ R_F + R_F^2]} \cdot \cos\theta$$

Where θ is the angle of displacement

From equation (2·24), $\tan\theta = \dfrac{1}{\sqrt{3}} \left(\dfrac{R_F - R_E}{R_E + R_F} \right)$

$$\text{now } \cos\theta = \frac{1}{\sqrt{[1 + \tan^2\theta]}} = \frac{1}{\sqrt{\left[1 + \frac{1}{3} \left(\frac{R_F - R_E}{R_E - R_F} \right)^2 \right]}}$$

$$= \frac{R_E + R_F}{\sqrt{\left[(R_E + R_F)^2 + \frac{1}{3} (R_F - R_E)^2 \right]}}$$

Therefore,

Torque producing component of I_D

$$= \frac{sV}{k_R} \cdot \sqrt{[R_E^2 + R_E\ R_F + R_F^2]} \times \frac{R_E + R_F}{\sqrt{\left[(R_E + R_F)^2 + \frac{1}{3} (R_F - R_E)^2 \right]}}$$

$$= \frac{sV}{k_R} \cdot \sqrt{[R_E^2 + R_E\ R_F + R_F^2]} \times \frac{R_E + R_F}{\sqrt{\left[\frac{4}{3} R_E^2 + \frac{4}{3} R_E R_F + \frac{4}{3} R_F^2 \right]}}$$

$$= \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_E + R_F) \tag{2·30}$$

Likewise, torque producing components of

$$I_E = \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_D + R_E) \tag{2·31}$$

$$\text{and } I_F = \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_D + R_E) \tag{2·32}$$

The set of equations (2·27), (2·28) and (2·29) enable the actual current per leg of the rotor circuits to be computed whilst (2·30), (2·31) and (2·32) give the torque producing components of these currents, if the slip s is known.

The steady slip can be computed by comparing the total torque component of the rotor currents with that of a balanced rotor operating against a similar load torque. If I_B is the steady rotor

current per phase under balanced conditions, then, from equations (2·30), (2·31) and (2·32)

$$3I_B = \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} \; (R_E + R_F + R_D + R_F + R_D + R_E)$$

$$\text{whence,} \quad s = \frac{\sqrt{3}.I_B}{V} \cdot \frac{k_R}{R_D + R_E + R_F}$$

Resistor Legs in Geometric Progression.

When the resistor legs are in G.P. the set of equations (2·27) to (2·32) can be simplified. If the common ratio of the G.P. is K and R_1, R_2 and R_3 represent the highest, intermediate and lowest total ohmic resistance per leg respectively, then

$$R_1 = KR_2, \; R_3 = \frac{R_2}{K} = \frac{R_1}{K^2}$$

Further, if I_1, I_2 and I_3 represent the currents in R_1, R_2 and R_3 respectively, from equation (2·27),

$$I_1 = \frac{sV}{k_R} \cdot \sqrt{[R_2^2 + R_2 \; R_3 + R_3^2]} = sV. \; \frac{\sqrt{[R_2^2 + R_2 \; R_3 + R_3^2]}}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

substituting in the above for R_1 and R_3.

$$I_1 = sV \; . \; \frac{\sqrt{\left[R_2^2 + R_2 \; \frac{R_2}{K} + \frac{R_2^2}{K^2} \right]}}{R_2 \frac{R_2}{K} + KR_2 \; \frac{R_2}{K} + KR_2 \; R_2} = \frac{sV}{R_2} \cdot \frac{1}{\sqrt{[1 + K + K^2]}} \qquad (2·33)$$

From (2·28),

$$I_2 = \frac{sV}{k_R} \; . \; \sqrt{[R_1^2 + R_1 \; R_3 + R_3^2]} = sV \; . \; \frac{\sqrt{[R_1^2 + R_1 \; R_3 + R_3^2]}}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

substituting for R_1 and R_3

$$I_2 = sV \; . \; \frac{\sqrt{\left[K^2 R_2^2 + KR_2 \; \frac{R_2}{K} + \frac{R_2^2}{K^2} \right]}}{\frac{R_2^2}{K} + R_2^2 + KR_2^2} = \frac{sV}{R_2} . \frac{\sqrt{[K^4 + K^2 + 1]}}{1 + K + K^2}$$

$$= \frac{sV}{R_2} \; . \; \sqrt{\left[\frac{1 - K + K^2}{1 + K + K^2} \right]} \qquad (2·34)$$

From (2·29)

$$I_3 = \frac{sV}{k_R} \cdot \sqrt{[R_1^2 + R_1 R_2 + R_2^2]} = sV \cdot \frac{\sqrt{[R_1^2 + R_1 R_2 + R_2^2]}}{R_2R_3 + R_1R_3 + R_1R_2}$$

substituting for R_1 and R_3

$$I_3 = sV \cdot \frac{\sqrt{[K^2R_2^2 + KR_2 R_2 + R_2^2]}}{\frac{R_2^2}{K} + R_2^2 + KR_2^2} = \frac{sV}{R_2} \cdot \frac{\sqrt{[K^4 + K^3 + K^2]}}{1 + K + K^2}$$

$$= \frac{sV}{R_2} \cdot \frac{K}{\sqrt{[1 + K + K^2]}} \qquad (2{\cdot}35)$$

Rotor I^2R Losses.

If R_B is the total rotor resistance per leg for balanced control and I_B the current per leg, then

$$\text{Total rotor } I^2R \text{ losses} = 3I_B{}^2 R_B = \frac{s^2V^2}{R_B} \qquad (2{\cdot}36)$$

For unbalanced control,

$$\text{Total rotor } I^2R \text{ losses} = I_1{}^2R_1 + I_2{}^2 R_2 + I_3{}^2 R_3$$

now, from equations (2·33), (2·34) and (2·35).

$$I_1 = \frac{sV}{R_2} \cdot \frac{1}{\sqrt{[1 + K + K^2]}}$$

$$I_2 = \frac{sV}{R_2} \cdot \sqrt{\left[\frac{1 - K + K^2}{1 + K + K^2}\right]}$$

and $$I_3 = \frac{sV}{R_2} \cdot \frac{K}{\sqrt{[1 + K + K^2]}}$$

Whence, upon substituting these in the above

Total rotor I^2R losses

$$= \frac{s^2 V^2}{R_2^2} \left\{ \frac{1}{1 + K + K^2} \cdot R_1 + \frac{1 - K + K^2}{1 + K + K^2} \cdot R_2 + \frac{K^2}{1 + K + K^2} \cdot R_3 \right\}$$

Substituting KR_2 for R_1 and $\frac{R_2}{K}$ for R_3 in the above

Total rotor I^2R losses

$$= \frac{s^2 V^2}{R_2^2} \cdot R_2 \left\{ \frac{K}{1 + K + K^2} + \frac{1 - K + K^2}{1 + K + K^2} + \frac{K}{1 + K + K^2} \right\}$$

$$=\frac{s^2 V^2}{R_2} \left\{ \frac{1+K+K^2}{1+K+K^2} \right\}$$

$$=s^2 \frac{V^2}{R_2} \tag{2·37}$$

Now for the same slip against similar load torques equations (2·36) and (2·37) should be equal, whence

$$s^2 \frac{V^2}{R_B} = s^2 \frac{V^2}{R_2}$$

whence $R_B = R_2$ (2·38)

The conclusion to be drawn from this is as follows :—If the total resistances of each of the three legs in an unbalanced rotor resistor are in geometrical progression, the starting characteristics obtained are similar to those of a balanced rotor resistor provided the ohmic resistance per leg of the balanced resistor is equal to the resistance of the unbalanced leg with the intermediate amount of ohmic resistance.

Unbalanced Rotor Current—Effective Total Torque.

From equations (2·30), (2·31) and (2·32), torque components of the unbalanced currents are :—

$$I_1 = \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_2 + R_3)$$

$$I_2 = \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_1 + R_3)$$

$$I_3 = \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_1 + R_2)$$

Total torque component $= I_1 + I_2 + I_3$

$$= \frac{sV}{k_R} \cdot \frac{\sqrt{3}}{2} (R_2 + R_3 + R_1 + R_3 + R_1 + R_2)$$

$$= sV \cdot \sqrt{3} \frac{(R_1 + R_2 + R_3)}{R_2R_3 + R_1R_3 + R_1R_2}$$

Substituting KR_2 for R_1 and $\frac{R_2}{K}$ for R_3 in the above

$$\text{Total torque component} = \sqrt{3}sV \frac{\left(KR_2 + R_2 + \dfrac{R_2}{K}\right)}{R_2 \dfrac{R_2}{K} + KR_2 \,.\, \dfrac{R_2}{K} + KR_2\ R_2}$$

$$= \frac{\sqrt{3}sV}{R_2} \frac{\left(K + 1 + \dfrac{1}{K}\right)}{\left(\dfrac{1}{K} + 1 + K\right)}$$

$$= \frac{\sqrt{3}sV}{R_2} \tag{2·39}$$

With balanced rotor current conditions,

$$\text{Total torque component of current} = 3I_B = \frac{3sV}{\sqrt{3}\,.\,R_2} = \frac{\sqrt{3}sV}{R_2} \tag{2·40}$$

Thus, from equations (2·39) and (2·40), for a balanced rotor resistor the three legs of which each have an ohmic resistance equal to the leg with the intermediate amount of ohmic resistance in the unbalanced rotor resistor, the torques are the same.

Peak Torques.

When notching up takes place on the controller, peak currents and torques will occur, just as in the case of the D.C. motor and balanced rotor current starting, previously considered.

If the resistance of the "middle" leg is R_x on notch x, then

Torque component of steady current from equation (2·40) $= \dfrac{\sqrt{3}sV}{R_x}$ on notch x.

On notch $x+1$ the resistance of the "middle" leg changes to R_{x+1} whence, torque component of steady current $= \dfrac{\sqrt{3}sV}{R_{x+1}}$

But since the three legs of the resistor are in G.P., with constant ratio, say, K, then $R_{x+1} = \dfrac{R_x}{K}$ whence.

$$\text{Torque component of peak current} = \frac{3sV}{R_x/K}$$

and the ratio of the peak/steady torque $= \dfrac{\sqrt{3sV}}{R_x/K} \cdot \dfrac{R_x}{\sqrt{3sV}} = K$ (2·41)

These peak torques and rotor currents occur at the instant of notch up and are usually of short duration, decreasing in value as the motor speed increases.

Slip.

For constant rotor current, the slip is directly proportional to the rotor ohmic resistance, so, if the resistance of the "middle" leg is R_x on notch x,

$$\text{Slip} \propto R_x$$

On notch $x+1$, the resistance of the "middle" leg is R_{x+1}. But, since the legs are in G.P., with constant ratio, say, K, then.

$$R_{x+1} = \frac{R_x}{K}$$

and the ratio of the slips on notches x and $x+1$ is $\dfrac{R_x}{R_x/K} = K$ (2·42)

Computation of K.

For equal peak currents on all notches, including the first,

$$K = \sqrt[n+1]{\frac{R_1}{r_m}}$$

For equal peaks on all notches, excluding the first,

$$K = \sqrt[n]{\frac{R_1}{r_m}}$$

Where, R_1 = Total resistance of the leg with the highest ohmic value.

r_m = Resistance per phase of the motor rotor winding only.

n = Total number of resistor sections.

It is convenient to base the calculations for an unbalanced rotor resistor on that of an equivalent balanced rotor. This connection is demonstrated in equation (2·38) which involves the use of R_2, then,

For equal peaks on all notches, including the first,

$$K = \sqrt[n+1]{\frac{R_1}{r_m}} = \sqrt[n+1]{\frac{R_2 \times K}{r_m}} = \sqrt[n]{\frac{R_2}{r_m}} \qquad (2·43)$$

For equal peaks on all notches, excluding the first,

$$K = \sqrt[n]{\frac{R_1}{r_m}} = \sqrt[n]{\frac{R_2 \times K}{r_m}} = \sqrt[n-1]{\frac{R_2}{r_m}} \tag{2·44}$$

If n has to be decided, and it is desired to have equal peak currents on all notches, including the first, equation (2·43) is modified thus,

For equal peak currents on all notches, including the first. n unknown,

$$K = \sqrt[n]{\frac{1}{s_{FL}}} \tag{2·45}$$

Where s_{FL} is the slip under normal full load conditions with the sliprings shorted. When K is thus calculated, R_2 can be found.

Example 11.

A 4-step unbalanced starting resistor is required for a 20 H.P. slipring induction motor. The equivalent of 100% rotor current must be passed on the first notch and equal rotor peak currents are required on the remainder. Standstill rotor volts between sliprings = 200, rotor full load current = 48. Full load slip = 5%.

$$\text{Resistance of rotor } r_m = \frac{0·05 \times 200}{48 \times \sqrt{3}} = 0·12 \text{ ohms per phase.}$$

For the equivalent of 100% rotor current passing on the first notch, equation (2·38) and subsequent conclusion may be applied.

$$\text{Whence } R_2 = \frac{200}{\sqrt{3} \times 48} = 2·4 \text{ ohms.}$$

$$\text{From equation (2·44) } K = \sqrt[4-1]{\frac{2·4}{0·12}} = 2·72$$

From which the various resistance values are

$2·4 \quad \times K = 6·5$
$R_2 \qquad = 2·4$
$2·4 \quad \div K = 0·883$
$0·883 \div K = 0·325$
$0·325 \div K = 0·120 = r_m$

From equations (2·33), (2·34) and (2·35), the steady rotor currents I_1, I_2 and I_3 on the first notch, assuming the motor to be starting against full load torque and consequently does not start are,
From (2·33)

$$I_1 = \frac{sV}{R_2} \cdot \frac{1}{\sqrt{[1 + K + K^2]}} = 1 \cdot \frac{200}{2·4} \cdot \frac{1}{\sqrt{[1 + 2·72 + 2·72^2]}}$$
$$= 25 \text{ amps}$$

From (2·34)

$$I_2 = \frac{sV}{R_2} \cdot \sqrt{\left[\frac{1 - K + K^2}{1 + K + K^2}\right]}$$

$$= 1 \cdot \frac{200}{2{\cdot}4} \cdot \sqrt{\left[\frac{1 - 2{\cdot}72 + 2{\cdot}72^2}{1 + 2{\cdot}72 + 2{\cdot}72^2}\right]} = 59{\cdot}5 \text{ amps}$$

From (2·35)

$$I_3 = \frac{sV}{R_2} \cdot \frac{K}{\sqrt{[1 + K + K^2]}} = K\ I_1 = 2{\cdot}72 \times 25 = 68 \text{ amps.}$$

On the second notch, the steady slip would be $1/K = 0{\cdot}368$, and on the third notch $0{\cdot}368/K = 0{\cdot}135$, etc. Upon working out the steady values of the currents as above, using the appropriate value of slip, the magnitudes of the currents will be found to be the same as above. These currents will alternate from section to section, lowest, intermediate and higher value of current obtaining on the highest, intermediate and lowest ohmic value legs respectively.

From equation (2·39) the total value of the torque component of rotor current $= \sqrt{3} \,.\, sV/R_2$. Substituting the values obtaining on the first notch, whence

$$\text{Total torque component of rotor current} = \frac{\sqrt{3}sV}{R_2} = \sqrt{3}.1.\frac{200}{2{\cdot}4}$$

$$= 144 \text{ amps.}$$

The total torque component of current for balanced rotor starting $= 3 \times 48 = 144$ amps.

The total torque component of the unbalanced rotor currents is therefore correct, the assumption being that the motor is starting against full load torque. This total torque component can be worked out from the conditions obtaining on any notch, provided the appropriate values of the slip and R_2 are used.

For the sake of completeness, and in order to illustrate fully the working, all the necessary details are tabulated and appended below. The rotor legs are nominated D, E and F.

Notch No.	Total Resistance Rotor Circuit/Leg			Rotor Steady Currents			Rotor Peak Current	Total Torque Component of Rotor Current	Slip
	D	E	F	D	E	F			
1	6·5	2·4	0·88	25	59·5	68	K times	144	1
2	0·325	2·4	0·88	68	25	59·5	steady	144	0·368
3	0·325	$0{\cdot}12 = r_m$	0·88	59·5	68	25	currents	144	0·135
4	$r_m = 0{\cdot}12$	$r_m = 0{\cdot}12$	$r_m = 0{\cdot}12$	48	48	48		144	0·05

Example 12.

An unbalanced starting resistor is required for a 50 H.P. slipring induction motor, the rotor peak currents being restricted to 175% of the rotor steady currents. What are the values of the total resistance per leg and steady rotor currents? Standstill rotor volts between sliprings = 400, full load rotor current = 60 amps. Full load slip = 5%. Assume the motor to be starting against full load torque.

This is a case of equal rotor peak currents on all notches.

Resistance of rotor $r_m = \dfrac{0{\cdot}05}{60} \times \dfrac{400}{\sqrt{3}} = 0{\cdot}192$ ohms per phase.

From equation (2.45) $K = \sqrt[n]{\dfrac{1}{s_{FL}}}$ i.e., $1{\cdot}75 = \sqrt[n]{\dfrac{1}{0{\cdot}05}}$

whence $n = 5{\cdot}35$.

Taking the next highest integer, $n = 6$, modified value of $K =$

$$\sqrt[6]{\frac{1}{0{\cdot}05}} = 1{\cdot}648$$

The equivalent value of R_2 to pass 1·648 times full load current with a balanced rotor current on the first notch, from equation (2·38), is

$$R_2 = \frac{400}{\sqrt{3} \times 60 \times 1{\cdot}648} = 2{\cdot}34 \text{ ohms}$$

From which the total values of the resistance are

$$\begin{aligned}
R_2 \times K &= 3{\cdot}84\\
R_2 &= 2{\cdot}34\\
2{\cdot}34 \div K &= 1{\cdot}42\\
1{\cdot}42 \div K &= 0{\cdot}864\\
0{\cdot}864 \div K &= 0{\cdot}524\\
0{\cdot}524 \div K &= 0{\cdot}318\\
0{\cdot}318 \div K &= 0{\cdot}192 = r_m
\end{aligned}$$

Nominating the legs by D, E, F, the resistance values on the various notches are :—

Notch	D	E	F
1	3·84	2·34	1·42
2	0·864	2·34	1·42
3	0·864	0·524	1·42
4	0·864	0·524	0·318
5	$0{\cdot}192 = r_m$	0·524	0·318
6	$0{\cdot}192 = r_m$	$0{\cdot}192 = r_m$	$0{\cdot}192 = r_m$

Based on the conditions obtaining on notch 1, $s = \dfrac{1}{1{\cdot}648} = 0{\cdot}606$

Steady value of rotor current in D leg $= s\ \dfrac{V}{R_2} \cdot \dfrac{1}{\sqrt{[1+K+K^2]}}$

$$= \frac{0{\cdot}606 \times 400}{2{\cdot}34} \times \frac{1}{\sqrt{[1+1{\cdot}648+1{\cdot}648^2]}} = 44{\cdot}8 \text{ amps}$$

Steady value of rotor current in E leg

$$= s\ \frac{V}{R_2} \cdot \sqrt{\left[\frac{1-K+K^2}{1+K+K^2}\right]} = \frac{0{\cdot}606 \times 400}{2{\cdot}34}$$

$$\times \sqrt{\left[\frac{1-1{\cdot}648+1{\cdot}648^2}{1+1{\cdot}648+1{\cdot}648^2}\right]} = 64{\cdot}3 \text{ amps}$$

Steady value of rotor current in F leg $= K \times D$ leg current $= 1{\cdot}648 \times 44{\cdot}8 = 74$ amps.

Total torque component of the unbalanced rotor currents based on conditions obtaining on notch 1,

From equation (2·39) total torque component of rotor currents =

$$\sqrt{3}\ \frac{sV}{R_2} = \sqrt{3} \times 0{\cdot}606 \times \frac{400}{2{\cdot}34} = 180 \text{ amps}$$

For balanced rotor currents, total torque component of current

$$= 3 \times 60 = 180 \text{ amps.}$$

A good check can be made by taking the geometric mean of the three unbalanced currents. In the case above.

$$\text{Geometric mean} = \sqrt[3]{44{\cdot}8 \times 64{\cdot}3 \times 74} = 59{\cdot}8 \text{ amps}$$

which agrees fairly well with the full load balanced rotor current per phase of 60 amps.

The values of the steady leg currents alternate from section to section, their magnitudes being 44·8, 64·3 and 74 amps. The peak values of these currents are 1·648 × steady values and the values of the steady slips on notches 1 to 6 are 0·606, 0·368, 0·223, 0·136, 0·0824 and 0·05 respectively.

Use of Relative Steady Rotor Currents.

From equation (2·33), $I_1 = \dfrac{sV}{R_2} \cdot \dfrac{1}{\sqrt{[1+K+K^2]}}$

$$\text{and } I_1 R_2 = s.V. \frac{1}{\sqrt{[1+K+K^2]}}$$

Now $R_1 = KR_2$, so that,

$$\text{Volts drop across leg } 1 = I_1R_1 = I_1KR_2 = s.V. \frac{K}{\sqrt{[1 + K + K^2]}}$$

Likewise, from equation (2·34),

$$\text{Volts drop across leg } 2 = I_2R_2 = s.V. \sqrt{\left[\frac{1 - K + K^2}{1 + K + K^2}\right]}$$

and, from equation (2·35),

$$\text{Volts drop across leg } 3 = I_3R_3 = I_3R_2/K = s.V. \frac{1}{\sqrt{[1 + K + K^2]}}$$

If these volts drops are plotted as ordinate against K as absissca, graphs are obtained relating the volts drop per leg and K.

For the purpose of drawing these graphs, V, the standstill rotor volts between slipring is assumed, a value of 100 volts being convenient. Thus, for a particular value of K, V is read from the graph and the relative steady rotor current is

$$\text{I relative} = \frac{\text{V relative (from graph)}}{\text{ohmic value of leg}}$$

From which,

$$= \quad \text{True value of current} = f \times s.\text{I relative}$$

where f is the ratio $\dfrac{\text{Actual Standstill rotor volts between sliprings}}{100}$

and s the slip.

The graphs of the volts drop will converge at a value of $100/\sqrt{3}$ on the ordinate at $K = 1$.

The use of these graphs greatly relieves the calculations of the steady rotor currents, but care should be taken to ensure that the correct curve is used, namely,

The uppermost curve corresponds to the leg with the highest ohmic value.

The centre curve corresponds to the leg with the intermediate ohmic value.

The lower curve corresponds to the leg with the lowest ohmic value.

SECTION 3—PRIMARY RESISTOR STARTERS.

Squirrel Cage Motors.

The squirrel cage motor is essentially the same as the slipring motor, but whereas the rotor windings of the slipring motor are brought out to three external sliprings, those of the squirrel cage motor are permanently connected by endrings, one at each end of the rotor. In effect, the rotor is isolated from any external connections. The rotor winding itself consists of heavy copper or brass bars, located in slots around the periphery of the rotor core. These bars are then all connected together by means of the above mentioned endrings. A squirrel cage motor is by virtue of its construction, cheaper and more robust than its slipring counterpart. These two assets are, however, offset by the fact that no direct connection can be made to the rotor to alter its characteristics. Four main methods are available for starting, viz. :—

(1) Direct Starting.

This method consists in applying the mains voltage directly to the stator windings, resulting in starting currents of the order of six to eight times full-load current. On account of this, this method is usually restricted to small motors of about 2 to 3 H.P. Special cases do exist where cage motors of 30 H.P., or more, are started in this manner, usually in mining machinery where complicated starting gear is both undesirable and vulnerable.

(2) Star/Delta Starting.

If the stator winding of the motor is delta connected for normal operation, a reduced stator voltage can be effected by connecting in star. This is a comparatively simple means of starting and one that is much used. At start, the stator windings are star connected and when the motor has run up to speed, the connections are changed to delta. The changeover from star/delta is accomplished by means of a special switch, which is usually fitted with mechanical inter-locks to ensure that the transition is rapidly made.

(3) Transformer Starting.

By this means various voltages can be applied to the stator windings to suit the starting conditions. Since the range of normal voltage reduction is comparatively small, auto-transformers are generally used.

(4) Primary Resistor Starting.

In this method the starting current is reduced by inserting external resistance in the stator windings.

This is shown schematically in Fig. (9).

Now $\text{H.P.} = \dfrac{2\pi n\text{T}}{33{,}000}$ Where n = r.p.m. and T = torque lbs. ft.

If P_{m} = Mechanical power developed in watts by the rotor at a speed n r.p.m. then $\dfrac{\text{P}_{\text{m}}}{746} = \dfrac{2\pi n\text{T}}{33{,}000}$

Whence, $\text{T} = \dfrac{\text{P}_{\text{m}} \times 33{,}000}{2\pi n \times 746} = \dfrac{\text{P}_{\text{m}} \times k_{\text{T}}}{n}$ where $k_{\text{T}} = \dfrac{33{,}000}{2\pi \times 746}$

Now, from equation (2·8), section 2 (*a*), $\text{P}_{\text{m}} = \text{P}_2\ (1-s)$

$$\text{Whence T} = k_{\text{T}} \ . \ \frac{\text{P}_2\ (1-s)}{n}$$

$$= k_{\text{T}} \ . \ \frac{\text{P}_2\ (1-s)}{n_1\ (1-s)}$$ Where n_1 = Synchronous speed of the rotor.

$$= k_{\text{T}} \ \frac{\text{P}_2}{n_1}$$

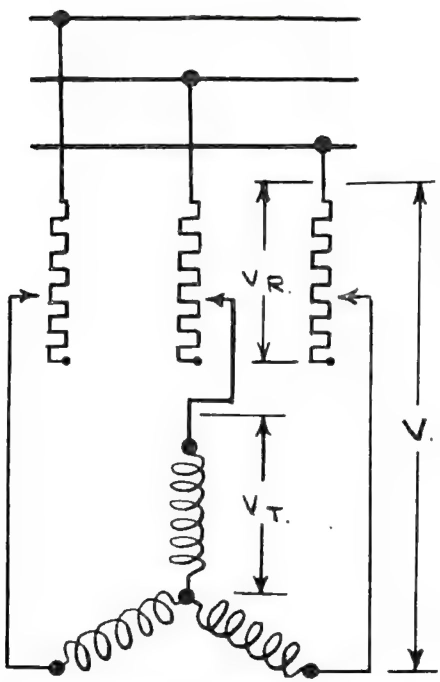


Fig. 9.—Primary starting resistor.

The torque is thus directly proportional to the rotor power input P_2. If the units of torque be taken as SYNCHRONOUS WATTS instead of the usual force × radius measure, then

$$T = P_2 \text{ Synchronous Watts} \tag{3·1}$$

The definition of the synchronous watt is that torque which would develop a power of 1 watt at the synchronous speed of the machine.

The utility of this unit lies in the fact that it involves the synchronous speed of the motor, which is a definite and known quantity.

From equation (2·9), section 2(*a*)

$$P_2 = \frac{\text{Rotor copper loss}}{\text{slip}}$$

$$= I_2^2 R_R^2/s$$

Where I_2 = rotor current/phase.
R_R = resistance of the rotor circuits per phase
s = fractional slip.

Then, from (3·1), the torque in synchronous watts/phase

$$T = I_2^2 \ R_R^2/s$$

Since it is necessary to obtain the starting torque in terms of the stator quantities, I_2 and R_R will have to be "referred" quantities. Now, the stator current is composed of two vectors, the rotor current and the magnetising current added vectorily. Assuming that $I_2^1 = 0·9\ I_1$, where I_2^1 is the rotor current referred to the stator, and letting the referred resistance of the rotor currents $= R_R^1$ then,

$T = (0·9\ I_1)^2 R_R^1/s = 0·81\ I_1^2\ R_R^1$ synchronous watts.

Thus, normal full load torque $T_{FL} = 0·81\ I_{FL}^2\ R_R^1/s_{FL} = kI_{FL}^2/s_{FL}$ (3·2)

Where I_{FL} and s_{FL} are the normal full load stator current and slip respectively.

If the stator current at starting is I_{SC}, then the starting torque

$T_S = kI_{SC}^2$ since the slip = 1 (3·3)

Thus, the ratio of starting to full load torque

$$= \frac{kI_{SC}^2}{kI_{FL}^2} \cdot s_{FL} = \left(\frac{I_{SC}}{I_{FL}} \right)^2 s_{FL} \tag{3·4}$$

For a motor with a short circuit current of six times full load, and a normal full load slip of five per cent., then the starting torque =

$$\left(\frac{6}{1} \right)^2 \times 0·05 = 1·8 \text{ times full load.}$$

Star/Delta Starting Torque.

For the above motor, $I_{sc} = \frac{6}{\sqrt{3}}$ times full load current, assuming the short circuit current to be directly proportional to the applied voltage.

Whence, the ratio of starting torque to full load torque =

$$\left(\frac{6}{\sqrt{3}}\right)^2 \times 0{\cdot}05 = 0{\cdot}6$$

Primary Resistor.

Let V = Applied voltage/phase.
V_T = Voltage at the motor terminals.
V_R = Volts drop across external resistance.
R_e = External resistance per phase.
r_{sc} = Total resistance per phase of stator and rotor in terms of stator quantities.
x_{sc} = Total reactance per phase of stator and rotor in terms of stator quantities.
Z_{sc} = Total impedance per phase of stator and rotor in terms of stator quantities.
I_{sc} = Starting current per phase.
$\cos\phi_{sc}$ = Short circuit power factor.

Then $Z_{sc} = \sqrt{[r_{sc}^2 + x_{sc}^2]}$

With an external resistance R_e inserted in the stator phases

$$Z_{sc} = \sqrt{[(R_e + r_{sc})^2 + x_{sc}^2]}$$

and

$$I_{sc} = \frac{V}{\sqrt{[(R_e + r_{sc})^2 + x_{sc}^2]}} \qquad (3{\cdot}5)$$

whence

$$R_e = \sqrt{\left[\left(\frac{V}{I_{sc}}\right)^2 - x_{sc}^2\right]} - r_{sc} \qquad (3{\cdot}6)$$

$$V_T = I_{sc} \cdot \sqrt{[r_{sc}^2 + x_{sc}^2]} = \frac{V \cdot \sqrt{[r_{sc}^2 + x_{sc}^2]}}{\sqrt{[(R_e + r_{sc})^2 + x_{sc}^2]}} \qquad (3{\cdot}7)$$

$$V_R = I_{sc} \times R_e = \frac{V\ R_e}{\sqrt{[(R_e + r_{sc})^2 + x_{sc}^2]}} \qquad (3{\cdot}8)$$

The set of equations (3·5) to (3·8) enable the starting current, etc., to be determined. Unfortunately, the values of the total reactance and resistance of the motor referred to the stator, will not be known in most cases. The value of the primary resistance

then involves the assumption of various quantities. These assumptions, provided they are made on a reasonable basis, should not introduce any serious defects.

Example 13.

A 5 H.P., 400 volts, 3 phase motor has a full load efficiency and power factor of 83% and 0·85 respectively. What would be the value of the external resistor to reduce the starting current to $2\frac{1}{2}$ times full load? Power factor on short circuit = 0·45 and the short circuit current is 5 times full load.

$$\text{Full load current of motor} = \frac{5 \times 746}{\sqrt{3} \times 400 \times 0{\cdot}83 \times 0{\cdot}85} = 7{\cdot}63 \text{ amps line}$$

Assuming the stator to be star connected :—

Total short circuit impedance per phase,

$$Z_{sc} = \frac{400}{\sqrt{3} \times 7{\cdot}63 \times 5} = 6{\cdot}06 \text{ ohms}$$

Total resistance per phase $= 6{\cdot}06 \times \cos\phi_{sc} = 6{\cdot}06 \times 0{\cdot}45 = 2{\cdot}72$ ohms

Total reactance per phase $= 6{\cdot}06 \times \sin\phi_{sc} = 6{\cdot}06 \,.\, \sqrt{[1 - \cos^2\phi_{sc}]}$
$= 5{\cdot}42$ ohms

Applying equation (3·6),

$$R_e = \sqrt{\left[\left(\frac{V}{I_{sc}}\right)^2 - x_{sc}^2\right]} - r_{sc}$$

$$= \sqrt{\left[\left(\frac{400}{3 \times 7{\cdot}63 \times 2{\cdot}5}\right)^2 - 5{\cdot}42^2\right]} - 2{\cdot}72$$

$$= 8{\cdot}12 \text{ ohms.}$$

Assuming the full load slip to be 5%, then the ratio of the

$$\text{starting to full load torque} = \left(\frac{2{\cdot}5}{1}\right)^2 \times 0{\cdot}05 = 0{\cdot}313$$

The computation of the necessary external resistance in the above example did not present any difficulties because all the necessary information was given. In cases where no information is available, the following may be taken as a rough guide for 400 volt, 3 phase, 50 cycle, squirrel cage induction motors.

H.P. of Motor	*Approximate Starting Current as a Percentage of Full Load Current at Full Voltage*
1/5	600
6/10	650/750
11/20	700/850
21/40	700/900

In general, the short circuit power factor for similar motors will vary from about 0·6 to 0·2, for the 1 to 40 H.P. motors respectively. It must be stressed, however, that these values are only intended as a rough guide for normal squirrel cage motors.

Delta Connected Stator.

The value of the external resistance R_e can be evaluated by equation (3·6) provided that r_{sc} and x_{sc} are calculated on the assumption that the stator is star connected.

Example 14.

What would be the external resistance required to limit the starting current to $3 \times$ full load for a 40 H.P., 400 volt, 50 cycle squirrel cage motor with a delta connected stator?

For a motor of this size, the full efficiency and power factor would be about 89% and 0·9 respectively.

$$\text{Whence, full load line current} = \frac{40 \times 746}{\sqrt{3} \times 400 \times 0{\cdot}89 \times 0{\cdot}9}$$

$$= 54 \text{ amps, say}$$

Assuming :—Short circuit current = 9 times full load.
Short circuit power factor = 0·2.

On the assumed basis of a star connected stator,

$$Z_{sc} = \frac{400}{\sqrt{3} \times 54 \times 9} = 0{\cdot}475 \text{ ohms.}$$

$$r_{sc} = 0{\cdot}475 \times \cos\phi_{sc} = 0{\cdot}095 \text{ ohms.}$$

$$x_{sc} = 0{\cdot}475 \times \sin\phi_{sc} = 0{\cdot}475.\ \sqrt{[1 - \cos^2\phi_{sc}]} = 0{\cdot}466 \text{ ohms.}$$

Apply equation (3·6).

$$R_e = \sqrt{\left[\left(\frac{V}{I_{sc}}\right)^2 - x^2_{sc}\right]} - r_{sc}$$

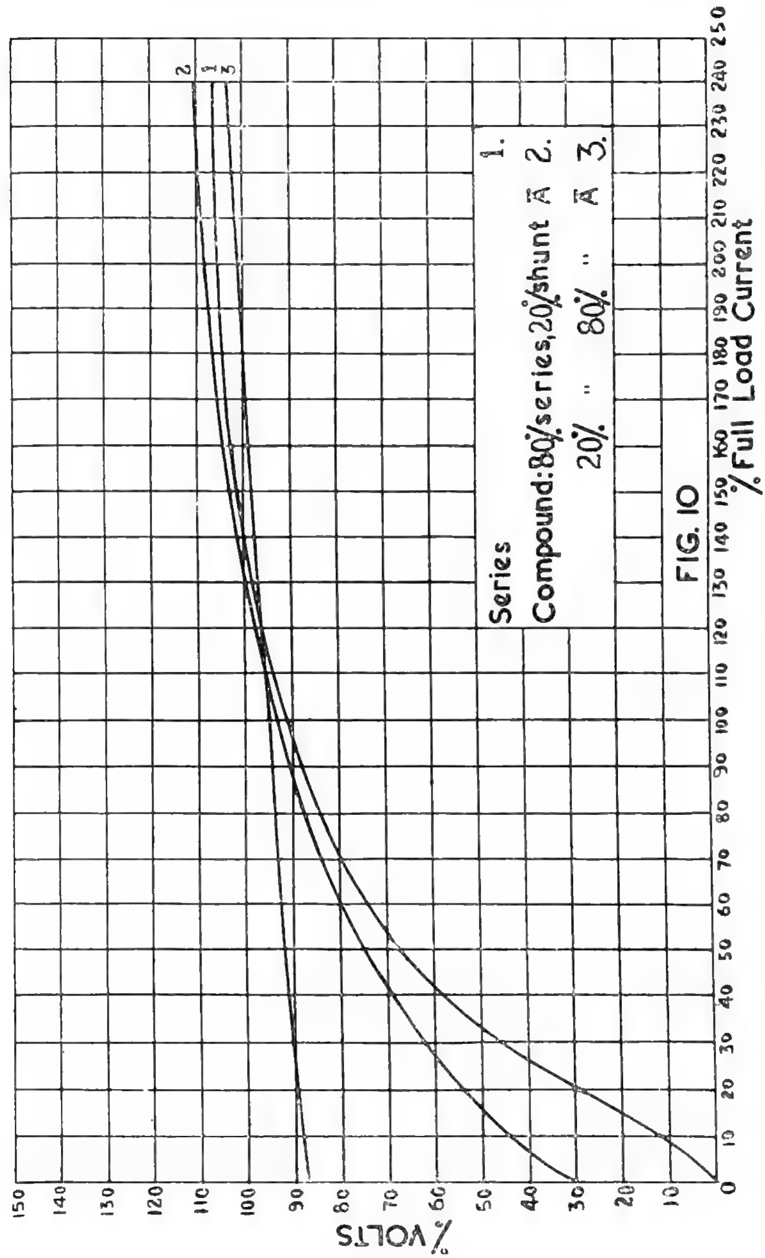
$$= \sqrt{\left[\left(\frac{400}{\sqrt{3} \times 54 \times 3}\right)^2 - 0{\cdot}466^2\right]} - 0{\cdot}095$$

$$= 1{\cdot}255 \text{ ohms.}$$

Assuming the full load slip = 3%, then the ratio of the starting to full load torque $= \left(\frac{3}{1}\right)^2 \times 0{\cdot}03 = 0{\cdot}27$

Comment on Starting Torques.

From the foregoing, it is evident that whilst starting currents can be reduced by reducing the voltage applied to the motor terminals, the starting torques obtained are comparatively poor. On the whole, this method of starting only finds applications where the starting torques required are 50% or less. By far the greater number of these are accomplished by Star/Delta or auto-transformer starting, primary resistor starting being but rarely used.



AVERAGE CHARACTERISTIC CURVES OF SERIES AND COMPOUND D.C. MOTORS.

TABLE I.

AVERAGE CHARACTERISTIC CURVES OF SERIES AND COMPOUND D.C. MOTORS.

CO-ORDINATES OF POINTS.

Armature Current as a Percentage of Full Load Armature Current	Back E.M.F. as a Percentage of Line Volts		
	Series Motor	Compound Motor. 80% Series, 20% Shunt Ampere Turns	Compound Motor. 20% Series, 80% Shunt Ampere Turns
0	0	30·6	87
10	12	44	88
20	30	54	89·5
30	45	62·5	90·5
40	58	69	91
50	67	74·6	91·5
60	74	80	92
70	79	83·5	93
80	84	87	93·5
90	88	90	94·5
100	91	93	95
120	96	97·5	96·5
140	100	101·5	98
160	102·5	104·5	99
180	104	106·5	100
200	105	108	101·5
220	106	109·5	102·5
240	106·5	110·5	103·5

The above characteristics are based on the following assumptions regarding the resistance of the motors.

SERIES MOTOR.

$$r_m = \frac{0{\cdot}09 \times V}{\text{Full Load Current}}$$

COMPOUND MOTOR—80% Series, 20% Shunt Ampere Turns.

$$r_m = \frac{0{\cdot}07 \times V}{\text{Full Load Current}}$$

COMPOUND MOTOR—20% Series, 80% Shunt Ampere Turns.

$$r_m = \frac{0{\cdot}05 \times V}{\text{Full Load Current}}$$

Where :— r_m = resistance of motor, ohms.
V = Line voltage.

RESISTANCE OF MOTOR WINDINGS.

The ohmic resistance of motor windings play a prominent part in starting resistor computations.

In the case of the A.C. slipring motor, the rotor loss, from equation (2·6) is $P_R = 3s\, E_2\, I_2 \,.\, \cos\phi_2$

When the motor is running under normal full load conditions, with the sliprings short circuited, then $3I_2^2\, r_m = 3s\, E_2\, I_2 \,.\, \cos\phi_2$. Under these conditions, $\cos\phi_2 \simeq 1$, from which the resistance of the motor windings per phase $r_m = \dfrac{sE_2}{I_2}$

Where, s = Normal full load slip.
E_2 = Rotor induced e.m.f. per phase at standstill.
I_2 = Normal full load rotor current per phase.

It should be noted that the term sE_2 is the voltage per phase required to circulate full load current in the rotor windings with the sliprings short circuited.

The armature resistance of D.C. machines can be expressed in a similar manner. In the case of the D.C. motor :

$$V = E_1 + I_A\, r_m$$

Where, V = Line volts.
E_1 = Back e.m.f.
I_A = Armature current.
r_m = Resistance of armature windings.

Thus, $r_m = \dfrac{V - E_1}{I_A}$

Now, if the back e.m.f. under normal full load conditions is expressed as a percentage of the terminal volts V,

Then, $r_m = \dfrac{V - f\%\, V}{I_A} = \left(1 - \dfrac{f}{100}\right)\dfrac{V}{I_A}$ Where I_A corresponds to the normal full load current of the armature.

For the average D.C. motor, values of f are :—
Shunt : $f = 95\%$, whence $r_m = 0{\cdot}05 \times V/I_A$
Series : $f = 91\%$, whence $r_m = 0{\cdot}09 \times V/I_A$

COMPOUND.

80% Series, 20% Shunt Ampere turns : $f = 93\%$, whence $r_m = 0{\cdot}07 \times V/I_A$

20% Series, 80% Shunt Ampere turns : $f = 95\%$, whence $r_m = 0{\cdot}05 \times V/I_A$

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5. Horse-Power of Leather Belts, etc.
6. Automobile Brakes (Axle Brakes).
7. Automobile Brakes (Transmission Brakes). } Connected
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to $5\frac{1}{2}$ inch diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, $5\frac{3}{4}$ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.
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